Homework 6

due November 12, 2003

1. Rosen 4.2 #6, pg. 142
For which integers c, $0 \leq c < 30$, does the congruence $12x \equiv c \pmod{30}$ have solutions? When there are solutions, how many incongruent solutions are there?

2. Rosen 4.2 #10, pg. 142
(a) Determine which integers a where $1 \leq a \leq 14$ have an inverse modulo 14.
(b) Find the inverse of each of the integers from (a) that have an inverse modulo 14.

3. Rosen 4.2 #15, pg. 143
Let p be an odd prime and k a positive integer. Show that the congruence $x^2 \equiv 1 \pmod{p^k}$ has exactly two incongruent solutions, namely $x \equiv \pm 1 \pmod{p^k}$.

4. Rosen 4.3 #7, pg. 149
A troop of 17 monkeys store their bananas in 11 piles of equal size with a twelfth pile of 6 left over. When they divide the bananas into 17 equal groups, none remain. What is the smallest number of bananas they can have?

5. Rosen 4.3 #15, pg. 150
Show that the system of congruences

$$\begin{aligned} x &\equiv a_1 \pmod{m_1} \\ x &\equiv a_2 \pmod{m_2} \end{aligned}$$

has a solution if and only if $(m_1, m_2) \mid (a_1 - a_2)$ (note that we are not assuming here that m_1 and m_2 are relatively prime!). Show that when there is a solution, it is unique modulo $[m_1, m_2]$. (*Hint*: Write the first congruence as $x = a_1 + km_1$ where k is an integer, and then insert this expression for x into the second congruence).

6. Rosen 6.1 #9, pg. 203
What is the remainder when 5^{100} is divided by 7?

7. Rosen 6.1 #34, pg. 203
Show that if p is a prime and $0 < k < p$, then $(p-k)!(k-1)! \equiv (-1)^k \pmod{p}$.