1. **Rosen 4.2 #6, pg. 142**
For which integers c, 0 ≤ c < 30, does the congruence 12x ≡ c (mod 30) have solutions? When there are solutions, how many incongruent solutions are there?

2. **Rosen 4.2 #10, pg. 142**
(a) Determine which integers a where 1 ≤ a ≤ 14 have an inverse modulo 14.
(b) Find the inverse of each of the integers from (a) that have an inverse modulo 14.

3. **Rosen 4.2 #15, pg. 143**
Let p be an odd prime and k a positive integer. Show that the congruence x² ≡ 1 (mod pᵏ) has exactly two incongruent solutions, namely x ≡ ±1 (mod pᵏ).

4. **Rosen 4.3 #7, pg. 149**
A troop of 17 monkeys store their bananas in 11 piles of equal size with a twelfth pile of 6 left over. When they divide the bananas into 17 equal groups, none remain. What is the smallest number of bananas they can have?

5. **Rosen 4.3 #15, pg. 150**
Show that the system of congruences
\[
\begin{align*}
x &\equiv a_1 \pmod{m_1} \\
x &\equiv a_2 \pmod{m_2}
\end{align*}
\]
has a solution if and only if \((m_1,m_2) | (a_1-a_2)\) (note that we are not assuming here that \(m_1\) and \(m_2\) are relatively prime!). Show that when there is a solution, it is unique modulo \([m_1,m_2]\). (*Hint: Write the first congruence as \(x = a_1 + km_1\) where \(k\) is an integer, and then insert this expression for \(x\) into the second congruence*).

6. **Rosen 6.1 #9, pg. 203**
What is the remainder when \(5^{100}\) is divided by 7?

7. **Rosen 6.1 #34, pg. 203**
Show that if \(p\) is a prime and \(0 < k < p\), then \((p-k)!(k-1)! (p) \equiv (-1)^k \pmod{p}\.\)