Homework Set Two: Fundamental Theorem of Algebra and Vector Spaces

Directions: Submit your solutions to the Calculational Exercises and the Proof-Writing Exercises at the beginning of lecture on Friday, October 9, 2009.

Calculational Exercises

1. Given any complex number $\alpha \in \mathbb{C}$, show that the coefficients of the polynomial $(z - \alpha)(z - \bar{\alpha})$ are real numbers.

2. Show that the space $V = \{ (x_1, x_2, x_3) \in \mathbb{F}^3 \mid x_1 + 2x_2 + 2x_3 = 0 \}$ forms a vector space.

3. Give an example of a nonempty subset $U \subset \mathbb{R}^2$ such that $U$ is closed under scalar multiplication but is not a subspace of $\mathbb{R}^2$.

Proof-Writing Exercises

1. Let $p(z)$ be a polynomial with real coefficients, and let $\alpha \in \mathbb{C}$ be a complex number. Prove that $p(\alpha) = 0$ if and only $p(\bar{\alpha}) = 0$.

2. Let $V$ be a vector space over $\mathbb{F}$. Then, given $a \in \mathbb{F}$ and $v \in V$ such that $av = 0$, prove that either $a = 0$ or $v = 0$.

3. Let $V$ be a vector space over $\mathbb{F}$, and suppose that $W_1$ and $W_2$ are subspaces of $V$. Prove that their intersection $W_1 \cap W_2$ is also a subspace of $V$. 