

Homework Set Two: Fundamental Theorem of Algebra and Vector Spaces

Directions: Submit your solutions to the Computational Exercises and the Proof-Writing Exercises at the **beginning** of lecture on **Friday, October 9, 2009**.

Computational Exercises

1. Given any complex number $\alpha \in \mathbb{C}$, show that the coefficients of the polynomial

$$(z - \alpha)(z - \bar{\alpha})$$

are real numbers.

2. Show that the space $V = \{(x_1, x_2, x_3) \in \mathbb{F}^3 \mid x_1 + 2x_2 + 2x_3 = 0\}$ forms a vector space.
3. Give an example of a nonempty subset $U \subset \mathbb{R}^2$ such that U is closed under scalar multiplication but is not a subspace of \mathbb{R}^2 .

Proof-Writing Exercises

1. Let $p(z)$ be a polynomial with real coefficients, and let $\alpha \in \mathbb{C}$ be a complex number. Prove that $p(\alpha) = 0$ if and only if $p(\bar{\alpha}) = 0$.
2. Let V be a vector space over \mathbb{F} . Then, given $a \in \mathbb{F}$ and $v \in V$ such that $av = 0$, prove that either $a = 0$ or $v = 0$.
3. Let V be a vector space over \mathbb{F} , and suppose that W_1 and W_2 are subspaces of V . Prove that their intersection $W_1 \cap W_2$ is also a subspace of V .