MAT 115A University of California Fall 2010

Homework 5

due November 3, 2010

1. Rosen 4.1 #22, pg. 150

Show by mathematical induction that if n is a positive integer, then $4^n \equiv 1 + 3n \pmod 9$.

2. Rosen 4.1 #24, pg. 150

Give a complete system of residues modulo 13 consisting entirely of odd integers.

3. Rosen 4.1 #28, pg. 150

Find the least positive residues modulo 47 of each of the following integer.

$$(a) \quad 2^{32} \qquad (b) \quad 2^{47} \qquad (c) \quad 2^{200}.$$

4. Rosen 4.1 #35, pg. 152

Show that for every positive integer m there are infinitely many Fibonacci numbers f_n such that m divides f_n. (*Hint:* Show that the sequence of least positive residues modulo m of the Fibonacci numbers is a repeating sequence.)

5. Rosen 4.2 #2 (a)-(c), pg. 156

Find all solutions of each of the following linear congruences.

$$(a) \quad 3x \equiv 2 \pmod 7$$
$$(b) \quad 6x \equiv 3 \pmod 9$$
$$(c) \quad 17x \equiv 14 \pmod{21}.$$

6. Rosen 4.2 #15, pg. 157

Let p be an odd prime and k a positive integer. Show that the congruence $x^2 \equiv 1 \pmod{p^k}$ has exactly two incongruent solutions, namely $x \equiv \pm 1 \pmod{p^k}$.