Homework 6

due November 10, 2010

1. Rosen 4.3 #7, pg. 164
A troop of 17 monkeys store their bananas in 11 piles of equal size with a twelfth pile of 6 left over. When they divide the bananas into 17 equal groups, none remain. What is the smallest number of bananas they can have?

2. Rosen 4.3 #15, pg. 165
Show that the system of congruences

$$x \equiv a_1 \pmod{m_1}$$
$$x \equiv a_2 \pmod{m_2}$$

has a solution if and only if $(m_1, m_2) \mid (a_1 - a_2)$ (note that we are not assuming here that m_1 and m_2 are relatively prime!). Show that when there is a solution, it is unique modulo $[m_1, m_2]$. (*Hint*: Write the first congruence as $x = a_1 + km_1$ where k is an integer, and then insert this expression for x into the second congruence).

3. Rosen 6.1 #9, pg. 221
What is the remainder when 5^{100} is divided by 7?

4. Rosen 6.1 #34, pg. 222
Show that if p is a prime and $0 < k < p$, then $(p-k)!(k-1)! \equiv (-1)^k \pmod{p}$.

5. Rosen 6.1 #40,41, pg. 222
Using the fact that p divides the binomial coefficient $\binom{p}{k}$ when $0 < k < p$, show that if a, b are integers then $(a+b)^p \equiv a^p + b^p \pmod{p}$. Use this to prove Fermat's little theorem.

6. Rosen 6.2 #2, pg. 231
Show that 45 is a pseudoprime to the bases 17 and 19.

7. Rosen 6.2 #20, pg. 232 (challenging!)
Show that if n is a Carmichael number then n is square-free.