1. Rosen 7.3 #11,13, pg. 266
Let $n$ be a positive integer. We say that $n$ is deficient if $\sigma(n) < 2n$ and we say that $n$ is abundant if $\sigma(n) > 2n$.
(a) Show that there are infinitely many deficient numbers.
(b) Show that there are infinitely many odd abundant numbers. (Hint: Look at integers of the form $n = 3^k \cdot 5 \cdot 7$).

2. Rosen 8.5 #2, pg. 321
Show that if $a_1, a_2, \ldots, a_n$ is a super-increasing sequence, then $a_j \geq 2^{j-1}$ for $j = 1, 2, \ldots, n$.

3. Rosen 8.5 #3, pg. 321
Show that the sequence $a_1, a_2, \ldots, a_n$ is super-increasing if $a_{j+1} > 2a_j$ for $j = 1, 2, \ldots, n - 1$.

4. Rosen 8.5 #10,12, pg. 322
A multiplicative knapsack problem is a problem of the following type: Given positive integers $a_1, a_2, \ldots, a_n$ and a positive integer $P$, find the subset, or subsets, of these integers with product $P$. Or equivalently, find all solutions of
\[ P = a_1^{x_1} a_2^{x_2} \cdots a_n^{x_n}, \]
where $x_j = 0$ or 1 for $j = 1, 2, \ldots, n$.
(a) Find all products of subsets of the integers 2, 3, 5, 6, 10 equal to 60.
(b) Show that if the integers $a_1, a_2, \ldots, a_n$ are pairwise relatively prime, then the multiplicative knapsack problem $P = a_1^{x_1} a_2^{x_2} \cdots a_n^{x_n}$, $x_j = 0$ or 1 for $j = 1, 2, \ldots, n$ is easily solved from the prime factorizations of the integers $P, a_1, a_2, \ldots, a_n$, and show that if there is a solution, then it is unique.