MAT 150A University of California Fall 2011

Homework 1
due September 30, 2011 in class

We will use Artin's numbering system so that "Artin 1.4.2" means Chapter 1, Section 4, Problem 2.

You are expected to hand in all problems. The reader will pick several problems at random for grading. To ensure credit for your homework you have to solve all of the problems!

Make sure your homework paper is legible, stapled and has your name clearly showing on each page. The homework is due on September 30 in class. **No late homeworks will be accepted!**

Read: Artin Chapters 1.3, 1.4, 2.1, 3.2

1. Heisenberg group: Let

$$H(F) = \{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \mid a, b, c \in F \}$$

be the Heisenberg group over the field F. Let

$$X = \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{and} \qquad Y = \begin{pmatrix} 1 & d & e \\ 0 & 1 & f \\ 0 & 0 & 1 \end{pmatrix}$$

be two elements in $H(F)$.

(1) Compute the matrix product XY and deduce that $H(F)$ is closed under matrix multiplication. Exhibit explicit matrices such that $XY \neq YX$ (which shows that $H(F)$ is always non-abelian).
(2) Find an explicit formula for the matrix inverse X^{-1} and deduce that $H(F)$ is closed under inverse.
(3) Prove the associative law for $H(F)$ and deduce that $H(F)$ is a group of order $|F|^3$ (do not assume that matrix multiplication is associative).
(4) Find the order of each element of the finite group $H(\mathbb{F}_2)$.
(5) Prove that every nonidentity element of the group $H(\mathbb{R})$ has infinite order.

2. Artin 1.3.4 (page 34)
3. Artin 1.4.1 (page 35)
4. Artin 1.4.5 (page 35)