Homework 3
due October 14, 2011 in class

1. Artin 2.3.10 (pg. 71)
2. Artin 2.4.3 (pg. 72)
3. Artin 2.4.8(a) (pg. 72)
4. Artin 2.4.10 (pg. 72)
5. Artin 2.4.13 (pg. 72)
6. Artin 2.4.17 (pg. 72)
7. If $\varphi : G \to H$ is an isomorphism, prove that $|\varphi(x)| = |x|$ for all $x \in G$. Deduce that any two isomorphic groups have the same number of elements of order n for each $n \in \mathbb{Z}^+$. Is the result true if φ is only assumed to be a homomorphism?
8. (a) Show that every subgroup of an abelian group is abelian.
   (b) Show by example that there exists a non-abelian group G such that every proper subgroup of G is abelian.