

## Homework 6

due November 9, 2011 in class

**Read:** Artin 4.5, 5.1

1. **Artin 4.5.1** pg. 148  
What is the matrix of the three-dimensional rotation through the angle  $\theta$  about the axis  $e_2$ ?
2. **Artin 4.5.4** pg. 148
  - (a) Prove that  $O_n$  and  $SO_n$  are subgroups of  $GL_n(\mathbb{R})$ , and determine the index of  $SO_n$  in  $O_n$ .
  - (b) Is  $O_2$  isomorphic to the product group  $SO_2 \times \{\pm I\}$ ? Is  $O_3$  isomorphic to  $SO_3 \times \{\pm I\}$ ?
3. **Artin 4.5.6** pg. 149  
Let  $A$  be a matrix in  $O_3$  whose determinant is  $-1$ . Prove that  $-1$  is an eigenvalue of  $A$ .
4. **Artin 4.5.13** pg. 149  
Prove that a rigid motion, as defined by (5.15), is bijective.
5. **Artin 5.1.3** pg. 188  
List all symmetries of the following figures.
  - (a) (1.4) (b) (1.5) (c) (1.6) (d) (1.7)
6. **Artin 5.1.4** pg. 188  
Let  $G$  be a finite group of rotations of the plane about the origin. Prove that  $G$  is cyclic.