Read: Artin Ch. 2.5, 2.6

1. Artin 2.6.8 (pg. 72)  
   Prove that the map $f: A \mapsto (A^t)^{-1}$ is an automorphism of $GL_n(\mathbb{R})$.

2. Prove that the kernel and image of a homomorphism are subgroups.

3. Find all subgroups of $S_3$, and determine which are normal.

4. Let $\varphi: G \to G'$ be a group homomorphism. Prove that $\varphi(x) = \varphi(y)$ if and only if $xy^{-1} \in \ker \varphi$.

5. (Artin 2.6.7 (pg. 72))
   (a) Let $H$ be a subgroup of $G$, and let $g$ be a fixed element of $G$. The conjugate subgroup $gHg^{-1}$ is defined to be the set of all conjugates $ghg^{-1}$, with $h \in H$. Prove that $gHg^{-1}$ is a subgroup of $G$.
   (b) Prove that a subgroup $H$ of $G$ is normal if and only if $gHg^{-1} = H$ for all $g \in G$.

6. Prove that the center of a group is a normal subgroup.

7. If $\varphi: G \to H$ is an isomorphism, prove that $|\varphi(x)| = |x|$ for all $x \in G$. Here $|x|$ denotes the order of the element $x$. Deduce that any two isomorphic groups have the same number of elements of order $n$ for each $n \in \mathbb{Z}^+$. Is the result true if $\varphi$ is only assumed to be a homomorphism?