1. **Rosen 4.1 #30, pg. 154**
Show by mathematical induction that if $n$ is a positive integer, then $4^n \equiv 1 + 3n \pmod{9}$.

2. **Rosen 4.1 #32, pg. 154**
Give a complete system of residues modulo 13 consisting entirely of odd integers.

3. **Rosen 4.1 #36, pg. 155**
Find the least positive residues modulo 47 of each of the following integer.

   $\begin{align*}
   (a) \quad 2^{32} & \quad (b) \quad 2^{17} & \quad (c) \quad 2^{200}.
   \end{align*}$

4. **Rosen 4.1 #43, pg. 156**
Show that for every positive integer $m$ there are infinitely many Fibonacci numbers $f_n$ such that $m$ divides $f_n$. (*Hint:* Show that the sequence of least positive residues modulo $m$ of the Fibonacci numbers is a repeating sequence.)

5. **Rosen 4.2 #2 (a)-(c), pg. 160**
Find all solutions of each of the following linear congruences.

   $\begin{align*}
   (a) \quad 3x & \equiv 2 \pmod{7} \\
   (b) \quad 6x & \equiv 3 \pmod{9} \\
   (c) \quad 17x & \equiv 14 \pmod{21}.
   \end{align*}$

6. **Rosen 4.2 #15, pg. 161**
Let $p$ be an odd prime and $k$ a positive integer. Show that the congruence $x^2 \equiv 1 \pmod{p^k}$ has exactly two incongruent solutions, namely $x \equiv \pm 1 \pmod{p^k}$. 

1