Homework 5
due November 6, 2015 in class

**Read:** Artin 2.11-2.12

1. Artin 2.11.6 (pg. 74)
   Let $G$ be a group containing normal subgroups of orders 3 and 5, respectively. Prove that $G$ contains an element of order 15.

2. Let $G$ be a finite group whose order is a product of two integers: $n = ab$. Let $H, K$ be subgroups of $G$ of orders $a$ and $b$ respectively. Assume that $H \cap K = \{1\}$. Prove that $HK = G$. Is $G$ isomorphic to the product group $H \times K$?

3. (a) Prove 2 has no inverse modulo 6.
   (b) Determine all integers $n$ such that 2 has an inverse modulo $n$.

4. Prove that the subset $H$ of $G = GL_n(\mathbb{R})$ of matrices whose determinant is positive forms a normal subgroup, and describe the quotient group $G/H$.

5. Prove that the subset $G \times 1$ of the product group $G \times G'$ is a normal subgroup isomorphic to $G$ and that $(G \times G')/(G \times 1)$ is isomorphic to $G'$.

6. Artin 2.M.2(a) (pg. 75)
   Prove that a group of even order contains an element of order 2.