Read: Artin 6.3, 6.4

1. Let $\text{Iso}(\mathbb{R}^n)$ be the group of isometries in $\mathbb{R}^n$. Prove that $O_n$ is not a normal subgroup of $\text{Iso}(\mathbb{R}^n)$.

2. Artin 6.3.2 pg. 188
   Let $m$ be an orientation-reversing motion. Prove that $m^2$ is a translation.

3. Let $SM$ denote the subset of orientation-preserving motions of the plane. Prove that $SM$ is a normal subgroup of $M := \text{Iso}(\mathbb{R}^2)$, and determine its index in $M$.

4. Find an isomorphism from the group $SM$ to the subgroup of $\text{GL}_2(\mathbb{C})$ of matrices of the form $\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, with $|a| = 1$.

5. Artin 6.4.2(a) pg. 188
   List all subgroups of the group $D_4$, and determine which ones are normal.

6. Artin 6.4.3 pg. 188
   (a) Compute the cosets of the subgroup $H = \{1, x^5\}$ in the dihedral group $D_{10}$ explicitly.
   (b) Prove that $H$ is normal and that $D_{10}/H$ is isomorphic to $D_5$.
   (c) Is $D_{10}$ isomorphic to $D_5 \times H$?