

Homework 7

due Friday November 20, 2015 in class

Read: Artin 6.3, 6.4

1. Let $\text{Iso}(\mathbb{R}^n)$ be the group of isometries in \mathbb{R}^n . Prove that O_n is not a normal subgroup of $\text{Iso}(\mathbb{R}^n)$.
2. **Artin 6.3.2** pg. 188
Let m be an orientation-reversing motion. Prove that m^2 is a translation.
3. Let SM denote the subset of orientation-preserving motions of the plane. Prove that SM is a normal subgroup of $M := \text{Iso}(\mathbb{R}^2)$, and determine its index in M .
4. Find an isomorphism from the group SM to the subgroup of $GL_2(\mathbb{C})$ of matrices of the form $\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, with $|a| = 1$.
5. **Artin 6.4.2(a)** pg. 188
List all subgroups of the group D_4 , and determine which ones are normal.
6. **Artin 6.4.3** pg. 188
 - (a) Compute the cosets of the subgroup $H = \{1, x^5\}$ in the dihedral group D_{10} explicitly.
 - (b) Prove that H is normal and that D_{10}/H is isomorphic to D_5 .
 - (c) Is D_{10} isomorphic to $D_5 \times H$?