1. Let $G$ be a discrete subgroup of $M := \text{Iso}(\mathbb{R}^2)$. Show that every subgroup of $G$ is discrete.

2. Prove that a discrete group $G$ consisting of rotations about the origin is cyclic and is generated by $\rho_\theta$ where $\theta$ is the smallest angle of rotation in $G$.

3. Let $G$ be a subgroup of $M$ which contains rotations about two different points. Prove algebraically that $G$ contains a translation.
   \textbf{Hint:} Write the two rotations as $t_a\rho_\theta$ and $t_b\rho_\eta$ and consider
   \[ (t_a\rho_\theta)(t_b\rho_\eta)(t_a\rho_\theta)^{-1}(t_b\rho_\eta)^{-1}. \]

4. Prove that every discrete subgroup of $O_2$ is finite.

5. A group $G$ acts \textbf{transitively} on a non-empty $G$-set $S$ if, for all $s_1, s_2 \in S$, there exists an element $g \in G$ such that $gs_1 = s_2$. Characterize transitive $G$-set actions in terms of orbits. Prove your answer.

6. A group $G$ acts \textbf{faithfully} on a $G$-set $S$ if $gs = s$ for all $s \in S$ implies $g = 1$. Show that $G$ acts faithfully on $S$ if and only if no two distinct elements of $G$ have the same action on every element of $S$. 

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\textbf{Reading:} Artin 6.5, 6.7