Read: Artin 6.8, 6.9, 7.1-7.8

(1) Artin 6.7.1. pg. 190
Let $G = D_4$ be the dihedral group of symmetries of the square.
(a) What is the stabilizer of a vertex? an edge?
(b) $G$ acts on the set of two elements consisting of the diagonal lines. What is the stabilizer of a diagonal?

(2) A map $\varphi : S \to S'$ of $G$-sets is called a homomorphism of $G$-sets if $\varphi(gs) = g\varphi(s)$ for all $s \in S$ and $g \in G$. Let $\varphi$ be such a homomorphism. Prove the following:
(a) The stabilizer $G_{\varphi(s)}$ contains the stabilizer $G_s$.
(b) The orbit of an element $s \in S$ maps onto the orbit of $\varphi(s)$.

(3) Find the number of distinguishable ways the edges of a square can be painted if six colors of paint are available and the same color may be used on more than one edge.

(4) Decide if the following statements are true or false. Briefly justify your response.
(a) Every $G$-set is also a group.
(b) Let $S$ be a $G$-set with $s_1, s_2 \in S$ and $g \in G$. If $gs_1 = gs_2$, then $s_1 = s_2$.
(c) Let $S$ be a $G$-set with $s \in S$ and $g_1, g_2 \in G$. If $g_1 s = g_2 s$, then $g_1 = g_2$.

(5) Artin 7.1.2. pg. 221
Let $H$ be a subgroup of a group $G$. Then $H$ operates on $G$ by left multiplication. Describe the orbits for this operation.

(6) Artin 7.2.7. pg. 221
Rule out as many of the following as possible as Class Equations for a group of order 10:
1+1+1+2+5, 1+2+2+5, 1+2+3+4, 1+1+2+2+2+2.

(7) Artin 7.7.4.(a) pg. 231
Prove that no group of order $pq$, where $p$ and $q$ are prime, is simple.

(8) Let $G$ be a finite group and let $P \leq G$ be a Sylow $p$-subgroup of $G$. Show that $P$ is the unique Sylow $p$-subgroup if and only if $P$ is normal in $G$. 