Homework 9

due December 4, 2015

Read: Artin 6.8, 6.9, 7.1-7.8

(1) **Artin 6.7.1.** pg. 190
Let $G = D_4$ be the dihedral group of symmetries of the square.
  (a) What is the stabilizer of a vertex? an edge?
  (b) G acts on the set of two elements consisting of the diagonal lines. What is the stabilizer of a diagonal?

(2) A map $\varphi : S \to S'$ of G-sets is called a *homomorphism* of G-sets if $\varphi(gs) = g\varphi(s)$ for all $s \in S$ and $g \in G$. Let φ be such a homomorphism. Prove the following:
  (a) The stabilizer $G_{\varphi(s)}$ contains the stabilizer G_s.
  (b) The orbit of an element $s \in S$ maps onto the orbit of $\varphi(s)$.

(3) Find the number of distinguishable ways the edges of a square can be painted if six colors of paint are available and the same color may be used on more than one edge.

(4) Decide if the following statements are **true** or **false**. Briefly justify your response.
  (a) Every G-set is also a group.
  (b) Let S be a G-set with $s_1, s_2 \in S$ and $g \in G$. If $gs_1 = gs_2$, then $s_1 = s_2$.
  (c) Let S be a G-set with $s \in S$ and $g_1, g_2 \in G$. If $g_1 s = g_2 s$, then $g_1 = g_2$.

(5) **Artin 7.1.2.** pg. 221
Let H be a subgroup of a group G. Then H operates on G by left multiplication. Describe the orbits for this operation.

(6) **Artin 7.2.7.** pg. 221
Rule out as many of the following as possible as Class Equations for a group of order 10:
1+1+1+2+5, 1+2+2+5, 1+2+3+4, 1+1+2+2+2+2.

(7) **Artin 7.7.4.(a)** pg. 231
Prove that no group of order pq, where p and q are prime, is simple.

(8) Let G be a finite group and let $P \leq G$ be a Sylow p-subgroup of G. Show that P is the unique Sylow p-subgroup if and only if P is normal in G.