MAT 150C University of California Spring 2005

Homework 2

due April 20, 2005 in class

We will use Artin's numbering system so that "Artin 11.1.9" means Chapter 11, Section 1, Problem 9.

(1) Artin 11.2.3 (pg. 442)
(2) Artin 11.2.7 (pg. 442)
(3) Artin 11.2.13 (pg. 442)
(4) Artin 11.2.19 (pg. 442)
(5) Artin 11.3.4 (pg. 443) – should say ...contains a nonzero integer
(6) (a) Show that $(2, x)$ is not a principal ideal in $\mathbb{Z}[x]$. This shows that $\mathbb{Z}[x]$ is not a P.I.D..
    (b) Show that $(2, x)$ is principal in $\mathbb{Q}[x]$. Which element generates $(2, x)$ in $\mathbb{Q}[x]$?
    (c) What is $(2, x)$ in $\mathbb{Z}/p\mathbb{Z}[x]$ where p is prime? For which p is $(2, x)$ maximal?
(7) (a) Let $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0 \in \mathbb{Z}[x]$. Suppose $r/s \in \mathbb{Q}$ is a root of $p(x)$ where r and s are coprime. Then $r \mid a_0$ and $s \mid a_n$.
    (b) Use part (a) to show that $x^3 - 3x - 1$ is irreducible in $\mathbb{Z}[x]$.