1. Stanley, Chapter 3.1
Let $G$ be a (finite) graph with vertices $v_1, \ldots, v_p$. Assume that some power of the probability matrix $M(G)$ has positive entries. (It is not hard to see that this is equivalent to $G$ being connected and containing at least one cycle of odd length, but you do not have to show this). Let $d_k$ denote the degree of vertex $v_k$. Let $D = d_1 + d_2 + \cdots + d_p = 2q - r$, where $G$ has $q$ edges and $r$ loops. Start at any vertex of $G$ and do a random walk on the vertices of $G$ as defined in the text. Let $p_k(\ell)$ denote the probability of ending up at vertex $v_k$ after $\ell$ steps. Assuming the Perron-Frobenius theorem, show that

$$\lim_{\ell \to \infty} p_k(\ell) = \frac{d_k}{D}.$$

The limiting probability distribution on the set of vertices of $G$ is called the **stationary distribution** of the random walk.

2. Stanley, Chapter 4.1
Draw Hasse diagrams of the 16 nonisomorphic four-element posets. (For a more interesting challenge, draw the 63 five-element posets – this part is not mandatory!).

3. Stanley, Chapter 4.2

(a) Let $P$ be a finite poset and $f: P \to P$ an order-preserving bijection, i.e., $f$ is a bijection (one-to-one and onto), and if $x \leq y$ in $P$ then $f(x) \leq f(y)$. Show that $f$ is an automorphism of $P$, that is, $f^{-1}$ is order-preserving.

(b) Show that the result of (a) need not be true of $P$ is infinite.