MAT 146 University of California Spring 2014

Homework 3

due Friday April 25, 2014 in class

1. Stanley, Chapter 3.1
Let G be a (finite) graph with vertices $v_1, \ldots, v_p$. Assume that some power of the probability matrix $M(G)$ has positive entries. (It is not hard to see that this is equivalent to G being connected and containing at least one cycle of odd length, but you do not have to show this). Let d_k denote the degree of vertex v_k. Let $D = d_1 + d_2 + \cdots + d_p = 2q - r$, where G has q edges and r loops. Start at any vertex of G and do a random walk on the vertices of G as defined in the text. Let $p_k(\ell)$ denote the probability of ending up at vertex v_k after ℓ steps. Assuming the Perron-Frobenius theorem, show that

$$\lim_{\ell \to \infty} p_k(\ell) = d_k/D.$$

The limiting probability distribution on the set of vertices of G is called the *stationary distribution* of the random walk.

2. Stanley, Chapter 4.1
Draw Hasse diagrams of the 16 nonisomorphic four-element posets. (For a more interesting challenge, draw the 63 five-element posets – this part is not mandatory!).

3. Stanley, Chapter 4.2

(a) Let P be a finite poset and $f\colon P \to P$ an order-preserving bijection, i.e., f is a bijection (one-to-one and onto), and if $x \leq y$ in P then $f(x) \leq f(y)$. Show that f is an automorphism of P, that is, f^{-1} is order-preserving.

(b) Show that the result of (a) need not be true of P is infinite.