1. Stanley, Chapter 5.5
A \((0,1)\)-necklace of length \(n\) and weight \(i\) is a circular arrangement of \(i\) 1’s and \(n-i\) 0’s. For instance, the \((0,1)\)-necklaces of length 6 and weight 3 are (writing a circular arrangement linearly) 000111, 001011, 010011, and 010101. (Cyclic shifts of a linear word represent the same necklace, e.g., 000111 is the same as 110001.) Let \(N_n\) denote the set of all \((0,1)\)-necklaces of length \(n\). Define a partial order on \(N_n\) by letting \(u \leq v\) if we can obtain \(v\) from \(u\) by changing some of the 0’s to 1’s. It is easy to see (you may assume it) that \(N_n\) is graded of rank \(n\), with the rank of a necklace being its weight.

1. Show that \(N_n\) is rank-symmetric, rank-unimodal, and Sperner.
   
   \textit{Hint:} Show that the \((0,1)\)-necklace poset is isomorphic to \(B_n/G\) for a suitable subgroup \(G\) of \(S_n\).

2. (not assigned, will be done in class, but if you can do this one yourself you are ready for research!) Show that \(N_n\) has a symmetric chain decomposition.

2. Stanley, Chapter 5.7
Let \(M\) be a finite multiset, say with \(a_i\) i’s for \(1 \leq i \leq k\). Let \(B_M\) denote the poset of all submultisets of \(M\), ordered by multiset inclusion. For instance, for \(a_1 = 2, a_2 = 3\) we have \(M = 11222\). Then the elements in \(B_M\) are \(\emptyset, 1, 2, 11, 12, 22, 112, 122, 222, 1122, 1222, 11222\) and for example \(122 \leq 1222\).

Use Theorem 5.8 to show that \(B_M\) is rank-symmetric, rank-unimodal, and Sperner. (There are other ways to do this problem, but you are asked to use Theorem 5.8. Thus you need to find a subgroup \(G\) of \(S_n\) for suitable \(n\) for which \(B_M \cong B_n/G\).)