(1) Let $a, b$ be elements of a field $F$, with $a \neq 0$. Prove that a polynomial $f(x) \in F[x]$ is irreducible if and only if $f(ax + b)$ is irreducible.

(2) Factor 30 into primes in $\mathbb{Z}[i]$.

(3) (Artin 12.5.5) Let $\pi$ be a Gauss prime. Prove that $\pi$ and $\bar{\pi}$ are associate if and only if either $\pi$ is associate to an integer prime or $\pi \bar{\pi} = 2$.

(4) (Artin 12.5.6) Let $R$ be the ring $\mathbb{Z}[\sqrt{3}]$. Prove that a prime integer $p$ is a prime element of $R$ if and only if the polynomial $x^2 - 3$ is irreducible in $\mathbb{F}_p[x]$.

(5) For the proof of Theorem 12.3.8 of Artin it is assumed that factorization exists in the polynomial ring $\mathbb{Z}[x]$. Explain why this is true.

(6) Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0 \in \mathbb{Z}[x]$ and let $p \in \mathbb{Z}$ be prime. Suppose that the coefficients of $f$ satisfy the following conditions:
   (a) $p$ does not divide $a_n$;
   (b) $p$ divides $a_{n-1}, \cdots, a_0$;
   (c) $p^2$ does not divide $a_0$.
   Show that $f(x)$ is irreducible in $\mathbb{Q}[x]$. If $f$ is primitive, it is irreducible in $\mathbb{Z}[x]$.

(7) Use Problem 6 to show that $x^4 + 10x + 5$ is irreducible in $\mathbb{Z}[x]$. Show that $x^n - p$ is irreducible in $\mathbb{Z}[x]$ for $n \geq 2$ and $p$ a prime integer. Is it possible to use Problem 6 to show that $x^4 + 1$ is irreducible? (Hint: Combine Problem 6 with Problem 1 with $a = b = 1$).