(1) For which fields $F$ and which primes $p$ does $x^p - x$ have a multiple root?

(2) Let $F$ be a field of characteristic $p$.
   (a) Apply Proposition 15.6.7 to the polynomial $x^p + 1$.
   (b) Factor this polynomial into irreducible factors in $F[x]$.

(3) (Artin 15.7.2) Determine the irreducible polynomial of each of the elements of $\mathbb{F}_8$ in the list 15.7.8.

(4) (Artin 15.7.7) Let $K$ be a finite field. Prove that the product of the nonzero elements of $K$ is $-1$.

(5) Prove that every element of $\mathbb{F}_p$ has exactly one $p$th root.

(6) (Artin 15.7.8) The polynomials $f(x) = x^3 + x + 1$ and $g(x) = x^3 + x^2 + 1$ are irreducible over $\mathbb{F}_2$. Let $K$ be the field extension obtained by adjoining a root of $f$, and let $L$ be the extension obtained by adjoining of $g$. Describe explicitly an isomorphism from $K$ to $L$, and determine the number of such isomorphisms.

(7) Determine the intermediate fields between $\mathbb{Q}$ and $\mathbb{Q}(\sqrt{2}, \sqrt{3})$.

Extra credit problem:
Use the Jordan Normal Form to prove the Spectral Theorem: every self-adjoint linear operator on a complex finite-dimensional vector space has real eigenvalues and there exists a basis with respect to which the matrix for this operator is diagonal.