MAT 261

University of California

Spring 2025

Homework 5 due June 4

Problem 1. Let $\alpha_1 = (1, -1)$ and $\alpha_2 = (0, 2)$ in \mathbb{R}^2 . Using properties 1-4 of an abstract root system discussed in class, construct all roots in the root system containing α_1 and α_2 .

Problem 2. Compute the Weyl group for the root system of Problem 1.

Problem 3. Let $\alpha_1 = (1, -1)$ and $\alpha_2 = (0, 1)$ in \mathbb{R}^2 . Using properties 1-4 of an abstract root system discussed in class, construct all roots in the root system containing α_1 and α_2 .

Problem 4. Compute the Weyl group for the root system of Problem 3. Show that this Weyl group is isomorphic to the Weyl group computed in Problem 2.

Problem 5. Show that the Weyl group of Problem 4 acts transitively on each length of root.

Problem 6. Let δ denote half the sum of the positive roots for a given root system. Show that for each positive simple root α

$$\frac{2(\alpha,\delta)}{(\alpha,\alpha)} = 1.$$