

Homework 3

due February 4, 2004

Question 1. Check that the following functions are harmonic (in some domain) and compute their harmonic conjugates

- (i) $u = 2x(1 - y)$
- (ii) $u = 2x - x^3 + 3xy^2$
- (iii) $u = \sinh x \sin y$
- (iv) $u = y/(x^2 + y^2)$

In each case use your answer to write down a function (in terms of z) analytic in some domain.

Question 2. Find a suitable branch cut for the function $f(z) = \sqrt{\frac{z-1}{z+1}}$. What can you say about this function for $|z|$ large?

Question 3. Let \log denote the principal branch of the logarithm. Evaluate

- (i) $\log(i)$ and $\log(i - 1)$
- (ii) $\log[i(i - 1)] - \log(i) - \log(i - 1)$

Question 4. Differentiate and give the appropriate region of analyticity for each of the following

- (i) $\log(z + 1)$
- (ii) $z^{(1+i)}$
- (iii) $\sqrt{z^2 - 2}$

Question 5. Prove that any function both analytic and antianalytic in any domain is constant.

Question 6. For what values of z is $\log z^2 = 2 \log z$ if the principal branch of the logarithm is used on both sides of the equation?