Homework 5

due February 18, 2004

This homework is slightly shorter than usual because of the midterm on February 13!

Question 1. Use Cauchy's integral formula, contour integration and the identity $2\cos(\theta) = z + 1/z$ for $z = \exp(i\theta)$ to show

$$\int_0^{2\pi} \frac{d\theta}{13 - 12\cos\theta} = \frac{2\pi}{5}\,.$$

Question 2. Let f be analytic on a region A and let γ be a closed curve in A. For any $z_0 \in A$ not on γ, show that

$$\int_\gamma \frac{f'(\zeta)}{\zeta - z_0} d\zeta = \int_\gamma \frac{f(\zeta)}{(\zeta - z_0)^2} d\zeta.$$

Can you think of a way to generalize this result?

Question 3. Suppose $\gamma : [a, b] \to \mathbb{C}$ is a curve and g is a continuous function defined along the image $\gamma([a, b])$. In class we showed that

$$G(z) = \frac{1}{2\pi i} \int_\gamma \frac{g(\zeta)}{\zeta - z} d\zeta$$

is analytic on $\mathbb{C}\backslash\gamma$. Show that the second derivative of $G(z)$ also exists and

$$G''(z) = \frac{1}{\pi i} \int_\gamma \frac{g(\zeta)}{(\zeta - z)^3} d\zeta.$$