This homework is slightly shorter than usual because of the midterm on February 13.

**Question 1.** Use Cauchy’s integral formula, contour integration and the identity

\[ 2 \cos(\theta) = z + 1/z \]  

for \( z = \exp(i\theta) \) to show

\[ \int_0^{2\pi} \frac{d\theta}{13 - 12 \cos \theta} = \frac{2\pi}{5}. \]

**Question 2.** Let \( f \) be analytic on a region \( A \) and let \( \gamma \) be a closed curve in \( A \). For any \( z_0 \in A \) not on \( \gamma \), show that

\[ \int_{\gamma} \frac{f'(\zeta)}{\zeta - z_0} d\zeta = \int_{\gamma} \frac{f(\zeta)}{(\zeta - z_0)^2} d\zeta. \]

Can you think of a way to generalize this result?

**Question 3.** Suppose \( \gamma : [a, b] \to \mathbb{C} \) is a curve and \( g \) is a continuous function defined along the image \( \gamma([a, b]) \). In class we showed that

\[ G(z) = \frac{1}{2\pi i} \int_{\gamma} \frac{g(\zeta)}{\zeta - z} d\zeta \]

is analytic on \( \mathbb{C} \setminus \gamma \). Show that the second derivative of \( G(z) \) also exists and

\[ G''(z) = \frac{1}{\pi i} \int_{\gamma} \frac{g(\zeta)}{(\zeta - z)^3} d\zeta. \]