Homework 7

due March 3, 2004

Question 1. If $\sum_{k=1}^{\infty} a_k$ converges, prove that $a_k \to 0$. If $\sum_{k=1}^{\infty} g_k(z)$ converges uniformly, show that $g_k \to 0$ uniformly.

Question 2. Show that $\sum_{n=1}^{\infty} \frac{1}{z^n}$ is analytic on $A = \{z \in \mathbb{C} \mid |z| > 1\}$.

Question 3. Compute a power series expansion for

$$f(z) = \frac{1}{1 - z - z^2} = \sum_{n=0}^{\infty} a_n z^n$$

to order z^6. Type the result for $\{a_0, a_1, a_2, a_3, a_4, a_5\}$ into

`http : //www.research.att.com/∼njas/sequences/index.html`

What do you find? (To reduce the number of entries note that a_6 through a_{11} are 13, 21, 34, 55, 89, 144.) Can you explain why?

Question 4. Show that

$$\sinh z = \sum_{n=1}^{\infty} \frac{z^{2n-1}}{(2n-1)!} \quad \text{and} \quad \cosh z = \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!}.$$

What is the radius of convergence in each case?

Question 5. Write the coefficients of the power series expansion of $\exp(z)$ about $z = 0$ as certain contour integrals. In turn, deduce the following integral identities

$$\int_0^{2\pi} d\theta\, e^{\cos(\theta)} \cos(\sin(\theta) - n\theta) = \frac{2\pi}{n!}$$

$$\int_0^{2\pi} d\theta\, e^{\cos(\theta)} \sin(\sin(\theta) - n\theta) = 0.$$