1. Biggs 11.4 # 5 page 114
Show that the number of derangements of \{1, 2, \ldots, n\} in which a given object (say 1) is in a 2-cycle is \((n - 1)d_{n-2}\). Hence construct a direct proof of the recursion formula
\[ d_n = (n - 1)(d_{n-1} + d_{n-2}) \quad \text{for } n \geq 3. \]

2. Biggs 11.5 # 4 page 118
Show that if \(1 \leq x \leq n\) then \(\gcd(x, n) = \gcd(n - x, n)\). Hence prove that the sum of all integers \(x\) which satisfy \(1 \leq x \leq n\) and \(\gcd(x, n) = 1\) is \(\frac{1}{2}n\Phi(n)\).

3. Biggs 11.8 # 6 page 124
Show that when \(n \geq m\)
\[ \binom{m}{m} + \binom{m+1}{m} + \cdots + \binom{n}{m} = \binom{n+1}{m+1}. \]

4. Biggs 12.5 # 3 page 136
Prove that if \(\pi\) and \(\tau\) are any members of \(S_n\) then \(\pi\tau\) and \(\tau\pi\) have the same type.

5. Biggs 12.7 # 11 page 141; mistake corrected
Use the sieve principle to show that the number of surjections from an \(n\)-set to a \(k\)-set is
\[ \sum_{i=0}^{k} (-1)^i \binom{k}{i} (k - i)^n. \]

6. Biggs 12.7 # 16 page 141
Show that the number of permutations in \(S_6\) of type \([1^4 2]\) is the same as the number of type \([2^3]\). If \(\alpha\) is of the first type, find the number of permutations \(\beta\) of the second type which satisfy \(\alpha\beta = \beta\alpha\).