MAT 149A University of California Winter 2006

Homework 2

due Wednesday January 25 in class

1. Biggs 11.4 # 5 page 114

Show that the number of derangements of $\{1, 2, \ldots, n\}$ in which a given object (say 1) is in a 2-cycle is $(n-1)d_{n-2}$. Hence construct a direct proof of the recursion formula

$$d_n = (n-1)(d_{n-1} + d_{n-2}) \quad \text{for } n \geq 3.$$

2. Biggs 11.5 # 4 page 118

Show that if $1 \leq x \leq n$ then $\gcd(x, n) = \gcd(n-x, n)$. Hence prove that the sum of all integers x which satisfy $1 \leq x \leq n$ and $\gcd(x, n) = 1$ is $\frac{1}{2}n\Phi(n)$.

3. Biggs 11.8 # 6 page 124

Show that when $n \geq m$

$$\binom{m}{m} + \binom{m+1}{m} + \cdots + \binom{n}{m} = \binom{n+1}{m+1}.$$

4. Biggs 12.5 # 3 page 136

Prove that if π and τ are any members of S_n then $\pi\tau$ and $\tau\pi$ have the same type.

5. Biggs 12.7 # 11 page 141; mistake corrected

Use the sieve principle to show that the number of surjections from an n-set to a k-set is

$$\sum_{i=0}^{k} (-1)^i \binom{k}{i} (k-i)^n.$$

6. Biggs 12.7 # 16 page 141

Show that the number of permutations in S_6 of type $[1^4 2]$ is the same as the number of type $[2^3]$. If α is of the first type, find the number of permutations β of the second type which satisfy $\alpha\beta = \beta\alpha$.