

## Homework 6

due Wednesday February 22 in class

**1. Biggs 20.7 # 2** page 272

Use the group  $G_\Delta$  to provide an example of the fact that if  $H$  and  $K$  are subgroups then  $H \cup K$  need not be a subgroup.

**2. Biggs 20.7 # 4** page 272

Let  $g$  be a given element of a group  $G$  and let  $C(g)$  denote the set of elements of  $G$  which commute with  $g$ , that is

$$C(g) = \{x \in G \mid xg = gx\}.$$

Show that  $C(g)$  is a subgroup of  $G$ . What is the relationship between these subgroups and the centre  $Z(G)$ ?

**3. Biggs 20.7 # 5** page 272

If  $G$  is the group of symmetries of the square, find  $C(g)$  for each  $g$  in  $G$ , and then find  $Z(G)$ .

**4. Biggs 20.8 # 3** page 276

The symmetry group of a regular pentagon is a group of order 10. Show that it has subgroups of each of the orders allowed by Lagrange's theorem, and sketch the lattice of subgroups.

**5. Biggs 21.1 # 4** page 283

List all symmetries of a regular pentagon, regarded as permutations of the corners 1,2,3,4,5, labelled in cyclic order.

**6. Biggs 21.1 # 5** page 283

**7. Biggs 21.2 # 2** page 284