MAT 149A University of California Winter 2006

Homework 7

due Wednesday March 1 in class

1. Biggs 21.2 # 5 page 285

Let G be a group of permutations of a set X and let k be an element of $G(x \to y)$. Prove that $G(x \to y)$ is equal to the right coset $G_y k$, and deduce that if u and v are any two elements in the same orbit if G then $|G_u| = |G_v|$.

2. Biggs 21.2 # 6 page 285

Let $X = \mathbb{Z}_5$ and suppose that G is the cyclic group of permutations of X generated by the permutation π defined by the rule $\pi(x) = 2x$. Write down the elements of G in cycle notation and determine the orbits of G on X.

3. Biggs 21.3 # 2 page 287

Let X denote the set of corners of a cube and let G denote the group of permutations of X which correspond to rotations of the cube. Show that:

(i) G has just one orbit on X;
(ii) if z is any corner, then $|G_z| = 3$;
(iii) $|G| = 24$.

4. Biggs 21.4 # 1 page 290

Show that there are just five different necklaces which can be constructed from five white beads and three black beads. Sketch them.

5. Biggs 21.4 # 3 page 290

6. Biggs 21.7 # 9 page 295

Show that 57 different cubes can be constructed by painting each face of a cube red, white, or blue.