

Some Common Mathematical Symbols and Abbreviations (with History)

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Binary Relations

$=$ (the **equals sign**) means “is the same as” and was first introduced in the 1557 book *The Whetstone of Witte* by Robert Recorde (c. 1510-1558). He wrote, “I will sette as I doe often in woorke use, a paire of parralles, or Gemowe lines of one lengthe, thus : $==$, bicause noe 2, thynges, can be moare equalle.” (Recorde used an elongated form of the modern equals sign.)

$<$ (the **less than sign**) mean “is strictly less than”, and $>$ (the **greater than sign**) means “is strictly greater than”. They first appeared in *Artis Analyticae Praxis ad Aequationes Algebraicas Resolvendas* (“The Analytical Arts Applied to Solving Algebraic Equations”) by Thomas Harriot (1560-1621), which was published posthumously in 1631.

Pierre Bouguer (1698-1758) later refined these to \leq (“less than or equals”) and \geq (“greater than or equals”) in 1734.

$:=$ (the **equal by definition sign**) means “is equal by definition to”. This is a common alternate form of the symbol “ $=_{\text{Def}}$ ”, which appears in the 1894 book *Logica Matematica* by the logician Cesare Burali-Forti (1861–1931). Other common alternate forms of the symbol “ $=_{\text{Def}}$ ” include “ $\stackrel{\text{def}}{=}$ ” and “ \equiv ”, the latter being especially common in applied mathematics.

Some Symbols from Mathematical Logic

\therefore (**three dots**) means “therefore” and first appeared in print in the 1659 book *Teusche Algebra* (“Teach Yourself Algebra”) by Johann Rahn (1622-1676).

\ni (the **such that sign**) means “under the condition that”. However, it is much more common (and less ambiguous) to just abbreviate “such that” as “s.t.”.

\Rightarrow (the **implies sign**) means “logically implies that”. (E.g., “if it’s raining, then it’s pouring” is equivalent to saying “it’s raining \Rightarrow it’s pouring.”)

The history of this symbol is unclear.

\Leftrightarrow (the **iff sign**) means “if and only if” and is used to connect logically equivalent statements. (E.g., “it’s raining iff it’s really humid” means simultaneously that “if it’s raining, then it’s

really humid” and that “if it’s really humid, then it’s raining”. In other words, the statement “it’s raining” implies the statement “it’s really humid” and vice versa.)

This notation “iff” is attributed to the great mathematician Paul R. Halmos (1916–2006).

\forall (the **universal quantifier** symbol) means “for all” and was first used in the 1935 publication *Untersuchungen ueber das logische Schliessen* (“Investigations on Logical Reasoning”) by Gerhard Gentzen (1909-1945). He called it the *All-Zeichen* (“all character”), in analogy with “ \exists ” (read: “there exists”).

\exists (the **existential quantifier**) means “there exists” and was first used in the 1897 book *Formulaire de mathematiquis* by Giuseppe Peano (1858-1932).

\square (the **Halmos tombstone**) means “QED”, which is an abbreviation for the Latin phrase *quod erat demonstrandum* (“which was to be proven”). “QED” has been the most common way to symbolize the end of a logical argument for many centuries, but the modern convention in mathematics is to use the “tombstone” in place of “QED”.

This “tombstone” notation is attributed to the great mathematician Paul R. Halmos (1916–2006).

Some Notation from Set Theory

\subset (the **is included in** sign) means “this set is a subset of” and \supset (the **includes** sign) means “this set has as a subset”. They were introduced in the 1890 book *Vorlesungen über die Algebra der Logik* (“Lectures on the Algebra of the Logic”) by Ernst Schröder (1841-1902).

\in (the **is in** sign) means “is an element of” and first appeared in the 1895 book *Formulaire de mathematiquis* by Giuseppe Peano (1858-1932). Peano originally used the Greek letter ϵ (which is the first letter of the Latin word *est*, meaning “is”), but it was Bertrand Russell (1872-1970) in his 1903 *Principles of Mathematics* that introduced the modern stylized version.

\cup (the **union** sign) means “take the elements that are in either set”, and \cap (the **intersection** sign) means “take the elements that the two sets have in common”. They were introduced in the 1888 book *Calcolo geometrico secondo l’Ausdehnungslehre di H. Grassmann preceduto dalle operazioni della logica deduttiva* (“Geometric Calculus based upon the teachings of H. Grassman, preceded by the operations of deductive logic”) by Giuseppe Peano (1858-1932).

\emptyset (the **null set** or **empty set** symbol) means “the set without any elements in it” and was first used in the 1939 book *Éléments de mathématique* by N. Bourbaki (a group of primarily European mathematicians—not a single person). It was borrowed simultaneously from the Norwegian, Danish and Faroese alphabets by group member André Weil (1906-1998).

∞ (**infinity**) denotes “a number of arbitrarily large magnitude” and first appeared in print in the 1655 book *De Sectionibus Conicis* (“On Conic Section”) by John Wallis (1616-1703).

Conjectured explanations for why Wallis used this symbol include its resemblance to the symbol *oo* used by Romans to denote the number 1000, its resemblance to the final letter of the Greek alphabet ω (and so is synonymous with being the “final” number), and the symbolism of the fact that one can traverse a given curve infinitely often.

Some Important Numbers in Mathematics

π (the **ratio of the circumference to the diameter of a circle**) denotes the number 3.141592653589..., and was first used by William Jones (1675-1749) in his 1706 book *Synopsis palmariorum mathesios* (“A New Introduction to the Mathematics”). However, it was Leonhard Euler (1707-1783) who first popularized the use of the letter π for this number in his 1748 book *Introductio in Analysin Infinitorum*.

Many people speculate that Jones chose the letter π because it’s the first letter in Greek word *perimetron* ($\pi\epsilon\rho\mu\epsilon\tau\rho\nu$), which roughly means “around”.

$e = \lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n$ (the **natural logarithm base**) denotes the number 2.718281828459..., and was first used by Leonhard Euler (1707-1783) in the manuscript *Meditatio in Experimenta explosione tormentorum nuper instituta* (“Meditation on experiments made recently on the firing of cannon”), written when he was just 21 years old. (Note that e is the first letter in *exponential*.)

The very famous mathematician Edmund Landau (1877-1938) once wrote that “The letter e may now no longer be used to denote anything other than this positive universal constant.”

$\gamma = \lim_{n \rightarrow \infty} (\sum_{k=1}^n \frac{1}{k} - \ln n)$ (the **Euler-Mascheroni constant**) denotes the number 0.577215664901..., and was first used by Lorenzo Mascheroni (1750-1800) in his 1792 *Adnotationes ad Euleri Calculum Integrale* (“Annotations to Euler’s Integral Calculus”).

The number γ is usually considered to be the third most important important non-basic number in mathematics, following closely π and e .

$i = \sqrt{-1}$ (the **imaginary unit**) was first used by Leonhard Euler (1707-1783) in his 1777 memoir *Institutionum calculi integralis* (“Foundations of Integral Calculus”).

Appendix: Some Common Latin Abbreviations and Phrases

(Cf. also http://en.wikipedia.org/wiki/List_of_Latin_phrases)

- i.e.** (*id est*) means “that is” or “in other words”. (It is used to paraphrase a statement that was just made, **not** to mean “for example”, and is **always** followed by a comma.)
- e.g.** (*exempli gratia*) means “for example”. (It is usually used to give an example of a statement that was just made and is **always** followed by a comma.)
- viz.** (*videlicet*) means “namely” or “more specifically”. (It is used to clarify a statement that was just made by providing more information and is **never** followed by a comma.)
- etc.** (*et cetera*) means “and so forth” or “and so on”. (It is used to suggest that the reader should infer further examples from a list has been started and is **usually not** followed by a comma.)
- et al.** (*et alii*) means “and others”. (It is used in place of listing multiple authors past the first and is **never** followed by a comma.) It’s also an abbreviation for *et alibi* means “and elsewhere”.

- cf.** (*conferre*) means “compare to” or “see also”. (It is used either to draw a comparison or to refer the reader to somewhere they can find more information and is **never** followed by a comma.)
- q.v.** (*quod vide*) means “which see” or “go look it up if you’re interested”. (It is used to cross-reference a different work or part of a work and is **never** followed by a comma. The plural form is “q.q.”)
- v.s.** (*vide supra*) means “see above”. (It is used to imply that more information can be found before the current point in a written work and is **never** followed by a comma.)
- N.B.** (*Nota Bene*) means “note well” or “pay attention to the following”. (It is used to imply that the wise reader will pay especially careful attention to the what follows and is **never** followed by a comma.)
- vs.** (*versus*) means “against” or “in contrast to”. (It is used to contrast two things and is **never** followed by a comma.)
- c.** (*circa*) means “around” or “near”. (It is used when giving an approximation, usually for a date, and is **never** followed by a comma. It’s also commonly written as “ca.,” “cir.,” or “circ.”)
- ex lib.** (*ex libris*) means “from the library of”. (It is used to indicate ownership of a book and is **never** followed by a comma.)
- *a fortiori* means “from the stronger” or “more importantly”.
 - *a priori* means “from before the fact” and refers to reasoning done before an event happens.
 - *a posteriori* means “from after the fact” and refers to reasoning done after an event happens.
 - *ad hoc* means “to this” and refers to reasoning that is quite specific to an event as it is happening. Such reasoning is usually considered to not generalize to other situations very well.
 - *ad infinitum* means “to infinity” or “without limit”.
 - *ad nauseam* means “causing sea-sickness” or “to excessive”.
 - *mutatis mutandis* means “changing what needs changing” or “with the necessary changes”.
 - *non sequitur* means “it does not follow” and refers to something that is out of place in a logical argument. (This is sometimes abbreviated as “non seq.”)
 - *Me transmittite sursum, Caledoni!* means “Beam me up, Scotty!”.
 - *Quid quid latine dictum sit, altum videtur* means “Anything said in Latin sounds profound”.