MAT 150B University of California Winter 2014

Homework 2

due Friday January 24, 2014 in class

1. Let $G = GL_n(\mathbb{R})$ and let $S = M_n(\mathbb{R})$ be the set of all $n \times n$ matrices over $\mathbb{R}$. Show that the map $G \times S \to S$ given by

$$(P, A) \mapsto (P^t)^{-1} A P^{-1}$$

defines an action of G on S.

2. Let $G = GL_n(\mathbb{C})$ and let $S = M_n(\mathbb{C})$ be the set of all $n \times n$ matrices over $\mathbb{C}$. Show that the map $G \times S \to S$ given by

$$(P, A) \mapsto (P^*)^{-1} A P^{-1}$$

defines an action of G on S.

3. Find the stabilizer of the identity matrix I_n under the action of $GL_n(\mathbb{R})$ on $M_n(\mathbb{R})$ given in Problem 1.

4. Find the stabilizer of the identity matrix I_n under the action of $GL_n(\mathbb{C})$ on $M_n(\mathbb{C})$ given in Problem 2.

5. Show that the product AA^* is hermitian for all $n \times m$ complex matrices A.

6. (Artin 8.3.1) Prove that if X^*AX is real for all complex vectors X, then A is hermitian.

7. (Artin 8.6.11) Prove that the eigenvectors associated to distinct eigenvalues of a hermitian matrix A are orthogonal.

8. (Artin 8.6.18) Use the Spectral Theorem to give a new proof of the fact that a positive definite real symmetric $n \times n$ matrix P has the form $P = AA^t$ for some $n \times n$ matrix A.