Homework 3

due January 31, 2014

1. Find a subgroup of $GL_2(\mathbb{R})$ which is isomorphic to $\mathbb{C}^\times$.

2. (Artin 9.1.2) A matrix P is orthogonal if and only if its columns form an orthonormal basis. Describe the properties that the columns of a matrix must have in order for it to be in the Lorentz group $O_{3,1}$.

3. (Artin 9.1.6) Prove that the following matrices are symplectic, if the blocks are $n \times n$:
$\begin{pmatrix} & -I \\ I & \end{pmatrix}$, $\begin{pmatrix} A^t & \\ & A^{-1} \end{pmatrix}$, $\begin{pmatrix} I & B \\ & I \end{pmatrix}$ where $B = B^t$ and A is invertible.

4. (Artin 9.3.1) Let P, Q be elements of SU_2, represented by the real vector (x_1, x_2, x_3, x_4), (y_1, y_2, y_3, y_4). Compute the real vector which corresponds to the product PQ.

5. Let G be the group of matrices of the form $\left(\begin{smallmatrix} x & y \\ & 1 \end{smallmatrix}\right)$, where $x, y \in \mathbb{R}$ and $x > 0$. Determine the conjugacy classes in G, and draw them in the (x, y)-plane.

6. Let $a = x_1 + ix_2$ and $b = x_3 + ix_4$ be two complex numbers. Show that $a\bar{a} + b\bar{b} = 1$ if and only if $x_1^2 + x_2^2 + x_3^2 + x_4^2 = 1$. Conclude that there is a bijective correspondence between $SU_2(\mathbb{C})$ and the 3-sphere S^3.