Homework 5

due February 14, 2014

1. Let ρ be a representation of a group G. Show that $\det \circ \rho$ is a one-dimensional representation.

2. Prove that the only one-dimensional representations of the symmetric group S_5 are the trivial representation defined by $\rho(g) = 1$ for all $g \in S_5$ and the sign representation.

3. Let $\{\cdot, \cdot\} \colon V \times V \to \mathbb{C}$ be a hermitian, positive definite form on the finite dimensional complex vector space V. In addition, let $\rho \colon G \to \mathrm{GL}(V)$ be a representation for the finite group G.

Show that then the form $\langle \cdot, \cdot \rangle \colon V \times V \to \mathbb{C}$ defined as

$$\langle v, w \rangle = \frac{1}{|G|} \sum_{g \in G} \{\rho_g(v), \rho_g(w)\} \qquad \text{for all } v, w \in V$$

is a hermitian, positive definite form.

4. Determine all irreducible representations of a cyclic group C_n.

5. (Artin 10.2.1) Prove that the standard three-dimensional representation of the tetrahedral group T is irreducible as a complex representation.