1. Prove or disprove: If an ideal $I$ contains a unit, then it is the unit ideal.

2. (Artin 11.2.1) For which integers $n$ does $x^2 + x + 1$ divide $x^4 + 3x^3 + x^2 + 6x + 10$ in $(\mathbb{Z}/n\mathbb{Z})[x]$?

3. Prove that in the ring $\mathbb{Z}[x]$, $(2) \cap (x) = (2x)$.

4. Is the set of polynomials $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$ such that $2^{k+1}$ divides $a_k$ an ideal in $\mathbb{Z}[x]$?

5. (Artin 11.3.2) Prove that a nonzero ideal $I \subseteq \mathbb{Z}[i]$ contains a nonzero integer.

6. (Artin 11.3.3 b) Find generators for the kernel of the map $\mathbb{R}[x] \to \mathbb{C}$ defined by $f(x) \mapsto f(2 + i)$.

7. (Artin 11.4.3 a and b) Describe each of the following rings:

   (a) $\mathbb{Z}[x]/(x^2 - 3, 2x + 4)$.

   (b) $\mathbb{Z}[i]/(2 + i)$.  