1. An inversion in $\pi = \pi_1\pi_2 \cdots \pi_n \in S_n$ in one-line notation is a pair $\pi_i, \pi_j$ such that $i < j$ and $\pi_i > \pi_j$. Let inv($\pi$) be the number of inversions of $\pi$.

   (a) Show that if $\pi$ can be written as a product of $k$ transpositions, then $k \equiv \text{inv}(\pi) \pmod{2}$.

   (b) Use part (a) to show that the sign of $\pi$ is well-defined.

2. Let $G$ act on $S$ with corresponding permutation representation $\mathbb{C}S$. Prove the following:

   (a) The matrices for the action of $G$ in the standard basis (meaning, with the elements of $S$ as the basis) are permutation matrices.

   (b) If the character of this representation is $\chi$ and $g \in G$, then $\chi(g)$ is the number of fixed points of $g$ acting on $S$.

3. **SAGE** exercise:

Write a SAGE program where you input $n$ (any positive integer) and a partition $\lambda$ of $n$, and the program returns the value of the character $\chi_{\text{def}}(\lambda)$ for the defining representation.