1. Let the group $G$ act on the set $S$. We say that $G$ acts \textit{transitively} on $S$ if, given any $s, t \in S$, there is a $g \in G$ such that $gs = t$. The group is \textit{doubly transitive} if, given any $s, t, u, v \in S$ with $s \neq u$ and $t \neq v$, there is a $g \in G$ with $gs = t$ and $gu = v$. Show the following:

(a) The orbits of the action of $G$ partition $S$.

(b) The multiplicity of the trivial representation in $V = \mathbb{C}S$ is the number of orbits. Thus if $G$ acts transitively, then the trivial representation occurs exactly once. What does this say about the module $M^\lambda$?

(c) If $G$ is doubly transitive and $V$ has character $\chi$, then $\chi - 1$ is an irreducible character of $G$.

\textit{Hint:} Fix $s \in S$ and use Frobenius reciprocity on the stabilizer $G_s \leq G$.

(d) Use part (c) to conclude that in $S_n$ the function

$$f(\pi) = (\text{number of fixed points of } \pi) - 1$$

is an irreducible character.

2. Show that every irreducible character of $S_n$ is an integer-valued function.