Homework 5

due February 13, 2015 for presentation in class

1. Let the group G act on the set S. We say that G acts *transitively* on S if, given any $s, t \in S$, there is a $g \in G$ such that $gs = t$. The group is *doubly transitive* if, given any $s, t, u, v \in S$ with $s \neq u$ and $t \neq v$, there is a $g \in G$ with $gs = t$ and $gu = v$. Show the following:

(a) The orbits of the action of G partition S.

(b) The multiplicity of the trivial representation in $V = \mathbb{C}S$ is the number of orbits. Thus if G acts transitively, then the trivial representation occurs exactly once. What does this say about the module M^λ?

(c) If G is doubly transitive and V has character χ, then $\chi - 1$ is an irreducible character of G.
Hint: Fix $s \in S$ and use Frobenius reciprocity on the stabilizer $G_s \leq G$.

(d) Use part (c) to conclude that in S_n the function

$$f(\pi) = (\text{number of fixed points of } \pi) - 1$$

is an irreducible character.

2. Show that every irreducible character of S_n is an integer-valued function.