MAT 180 University of California Winter 2015

Homework 8

due March 13, 2015 for presentation in class

1. Fix a partition λ and fix an ordering of standard λ-tableaux $t_1, t_2, \ldots$. Define the axial distance from k to $k+1$ in tableau t_i to be

$$\delta_i = \delta_i(k, k+1) = (c' - r') - (c - r),$$

where c, c' and r, r' are the column and row coordinates of k and $k+1$, respectively, in t_i. Young's seminormal form assigns to each transposition $s_k = (k, k+1)$ the matrix $\rho_\lambda(s_k)$ with entries

$$\rho_\lambda(s_k)_{i,j} = \begin{cases} 1/\delta_i & \text{if } i = j, \\ 1 - 1/\delta_i^2 & \text{if } s_k t_i = t_j \text{ and } i < j, \\ 1 & \text{if } s_k t_i = t_j \text{ and } i > j, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that every row and column of $\rho_\lambda(s_k)$ has at most two nonzero entries.

(b) Show that ρ_λ can be extended to a representation of S_n, where λ is a partition of n, by using the relations in Homework 6 #2.

(c) Show that this representation is equivalent to the one afforded by $\mathcal{S}^\lambda$.

2. Prove the following results in two ways: once using representation theory and once using combinatorics.

(a) If $K_{\lambda\mu} \neq 0$, then $\mu \trianglelefteq \lambda$.

(b) Suppose μ and ν are compositions with the same parts (only rearranged). Then for any λ, $K_{\lambda\mu} = K_{\lambda\nu}$.
Hint: For the combinatorial proof, consider the case where μ and ν differ by an adjacent transposition of parts.