Problem 1.

(1) Recall from class that

\[ h_n = \sum_{\lambda \vdash n} \frac{1}{z_\lambda} p_\lambda, \]

where \( z_\lambda = \prod_i i^{m_i} m_i! \) for \( \lambda = (1^{m_1}, 2^{m_2}, \ldots) \). Show that this is equivalent to Newton’s determinant formula

\[
h_n = \frac{1}{n!} \det \begin{pmatrix} p_1 & -1 & 0 & \ldots & 0 \\ p_2 & p_1 & -2 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{n-1} & p_{n-2} & \ldots & \ldots & -(n-1) \\ p_n & p_{n-1} & \ldots & \ldots & p_1 \end{pmatrix}
\]

(2) Show that \( e_n \) is given by the same determinant without the minus signs.

Problem 2. Prove the identity

\[
s_{(n-1,n-2,\ldots,1)}(x_1, \ldots, x_n) = \prod_{1 \leq i < j \leq n} (x_i + x_j).
\]

Problem 3. Permutation \( \pi = x_1 x_2 \cdots x_n \) has a descent at index \( i \) if \( x_i > x_{i+1} \). The corresponding descent set is

\[ \text{Des } \pi = \{ i \mid i \text{ is a descent of } \pi \}. \]

Similarly, \( i \) is a descent for standard tableau \( T \) if \( i + 1 \) is in a lower row than \( i \) in \( T \) (in English notation). The descent set of \( T \) is

\[ \text{Des } T = \{ i \mid i \text{ is a descent of } T \}. \]

(1) Show that if, by Robinson-Schensted, \( Q(\pi) = Q \), then \( \text{Des } \pi = \text{Des } Q \).

(2) Let \( \lambda \vdash n, S = \{ n_1 < n_2 < \cdots < n_k \} \subset \{ 1, 2, \ldots, n-1 \} \), and \( \mu = (n_1, n_2 - n_1, \ldots, n - n_k) \). Then

\[
|\{ \pi \in S_n \mid \text{shape}(Q(\pi)) = \lambda, \text{Des } \pi \subset S \}| = f^\lambda K_{\lambda \mu}.
\]
Problem 4. Let $T$ be a standard Young tableau of shape $\lambda$, and let $b$ be its corner box occupied by entry 1. Define
\[ \Delta(T) = \text{jdt}_b(\tilde{T}) \]
where $\tilde{T}$ is the skew standard tableau of shape $\lambda/(1)$ obtained from $T$ by removing the box $b$ and subsequently decreasing all the remaining entries by one. The evacuation tableau $\text{evac}(T)$ is the standard tableau of shape $\lambda$ encoded by the sequence of shapes of
\[ \emptyset, \Delta^{n-1}(T), \Delta^{n-2}(T), \ldots, \Delta^2(T), \Delta(T), T. \]

For a permutation $\pi = x_1 x_2 \cdots x_n \in S_n$ define
\[ w^\# = (n+1-x_n) (n+1-x_{n-1}) \cdots (n+1-x_1). \]

(1) Show: If $w$ is mapped to $(P, Q)$ under RSK, then $w^\#$ is mapped to $(\text{evac}(P), \text{evac}(Q))$ under RSK.
(2) Show that $\text{evac}$ is an involution.