Exercise 1: Translate the following sentences into symbolic sentences with quantifiers, using the universe of all things:

(i) There is at least one dog with one tail.
(ii) Every complex number is the root of some polynomial with real coefficients.

Think about whether each statement is true or false. Find several equivalent denials of each in symbolic sentences.

Exercise 2: Which of the following quantified sentences are true? The universe is given in brackets.

\[(i) \, (\exists! z)(\forall r)(z + r = r) \quad [\mathbb{R}] \]
\[(ii) \, (\exists z)(z^2 + 1 = 0) \quad [\mathbb{R}] \]
\[(iii) \, (\exists z)(z^2 + 1 = 0) \quad [\mathbb{C}] \]
\[(iv) \, (\forall n)(\exists k)(k > n) \quad [\mathbb{N}] \]

Exercise 3: For each of the following two cases, find a subset \( A \) of \( \mathbb{R} \) such that, when used as a universe, the symbolic statement:

\[(\forall x)(\exists! y)(y^2 = x)\]

is (a) true; (b) false.

* Exercise 4: Prove the following statements directly.

(i) Let \( n \) be a real number. If \( n \) is a natural number, then \( \frac{n(n + 1)}{2} \) is also a natural number.

(ii) Let \( n \) be a natural number. If \( n \) is an odd prime, then either \( n \) has remainder 1 when divided by 4, or \( n \) has remainder 3 when divided by 4.