

**Math 127C Homework 1 (first part), Spring 2021**

Due: Friday, May 12

- (1) (Triangle Inequality) [Exercise 1.1.(b)] Prove that

$$\|x + y\| \leq \|x\| + \|y\|.$$

[*Hint:* Compute  $\langle x + y, x + y \rangle$  and apply the Cauchy-Schwarz inequality which says that  $\langle x, y \rangle \leq \|x\| \|y\|$ .]

- (2) (Matrix supremum norm)[Exercise 1.2] If  $A$  is an  $r$  by  $m$  matrix and  $B$  is an  $m$  by  $c$  matrix show that

$$|AB| \leq m|A||B|.$$

- (3) (Theorem 18.3) Find a shortest sequence of type (2) and type (3) elementary row operations which have the effect of switching the first two rows of a matrix. Show that there is no such sequence using only type (2) operations.
- (4) (Theorem 1.6) Prove that if  $B$  is the matrix obtained by applying an elementary row operation to  $A$ , then

$$\text{rank } B = \text{rank } A.$$