Math 127C Homework 2, Spring 2024

Due: Friday, May 26 but relevant for the May 19 midterm

(1) (Transitivity)

Assume that $A \subseteq B \subseteq C$ are metric spaces.

(a) Show that if A is open in C then A is open in B.

(b) Find an example in which A is open in B but A is not open in C.

(2) Assume that A and B are compact subsets of a metric space.

(a) Show that their union $A \cup B$ is compact.

(b) Show that their intersection $A \cap B$ is compact.

(3) (Linearity)[Exercise 2.5.1]

Assume that $\mathbb{R}^c \supseteq A \xrightarrow{f} \mathbb{R}^r$, $a \in A$ and $c \in \mathbb{R}$. Show that if $f'(\mathbf{a}; \mathbf{u})$ exists then $f'(\mathbf{a}; c\mathbf{u})$ exists and $f'(\mathbf{a}; c\mathbf{u}) = cf'(\mathbf{a}; \mathbf{u})$.

(4) (Computation)[Exercise 2.5.2]

Assume that $\mathbb{R}^2 \xrightarrow{f} \mathbb{R}$ is given by $f((0,0)) = 0$ and otherwise $f((x,y)) = \frac{xy}{x^2+y^2}$.

(a) For which vectors $\mathbf{u} \in \mathbb{R}^2 - \{(0,0)\}$ does $f'((0,0), \mathbf{u})$ exist? Evaluate it when it exists.

(b) Do $D_1 f$ and $D_2 f$ exist at $(0,0)$?

(c) Is f differentiable at $(0,0)$?

(d) Is f continuous at $(0,0)$?