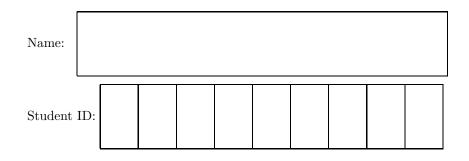
Math 127C Midterm I Friday, April 19 Spring 2024



Choose any five of the below problems to solve.

You may not use a calculator.

You may use one page of notes.

You may not use the textbook.

1. (20 pts Continuity)

Consider the function  $f : \mathbb{R}^4 \to \mathbb{R}$  defined by  $f((w, x, y, z)) = \langle (w, x), (y, z) \rangle$ . For any  $\epsilon > 0$  find  $\delta > 0$  (which will depend on  $\epsilon$ ) so that if  $|(w, x, y, z)| < \delta$  then  $f((w, x, y, z)) < \epsilon$ . This shows that the function f is continuous. 2. (20 pts Matrices) There is a sequence of elementary row operations which transforms the matrix \_\_\_\_\_

2	0	2	0	
1	2	0	1	
0	1	2	0 1 0	

to the reduced row eschelon form

[1]	0	0	a	1
0	1	0	b	.
$\left[\begin{array}{c}1\\0\\0\end{array}\right]$	0	1	c	

Find (a, b, c).

3. (20 pts Topology) Prove either of the following two equivalent facts:

$$\left\{ \left[ \begin{array}{c} a \\ b \end{array} \right] | \langle (a,b), (2,3) \rangle \neq 0 \right\}$$

is open in  $\mathbb{R}^2$ .

(b)

(a)

$$\left\{ \left[ \begin{array}{c} a \\ b \end{array} \right] | \langle (a,b), (2,3) \rangle = 0 \right\}$$

is closed in  $\mathbb{R}^2$ .

4. (20 pts Compacta) Consider a continuous function  $f : \mathbb{R}^2 \to \mathbb{R}$ . Assume that if |(x, y)| > 10 then f(x, y) = 0. Show that there is some point in  $\mathbb{R}^2$  at which f takes its maximum value. Note that  $\mathbb{R}^2$  is not compact. 5. (20 pts Derivatives)

Consider the function  $f: \mathbb{R}^2 \to \mathbb{R}$  with f((0,0)) = 0 and otherwise

$$f((x,y)) = \frac{xy}{x^2 + y^2}.$$

This function is differentiable at every point except (0,0) where it is not. Find the following or show they do not exist:

- (a) the two by one total derivative matrix (Df)(1,2),
- (b) f'((0,0);(1,2)),
- (c) the two by one Jacobian matrix Jacf((0,0)).

6. (20 pts Class  $C^1)$  Consider the function  $f:\mathbb{R}^2\to\mathbb{R}^2$  given by

$$f((x,y)) = \left[ egin{array}{c} xy \\ x+y \end{array} 
ight].$$

- (a) Compute all the directional derivatives of f.
- (b) Use part (a) to show that f is continuously differentiable. Recall the definition: A function is continuously differentiable if all parial derivatives  $D_j f_i$  exist and are continuous.