Math 127C Midterm I Friday, April 19 Spring 2024

Name:	

Student ID:									

Choose any five of the below problems to solve.
You may not use a calculator.
You may use one page of notes.
You may not use the textbook.

1. (20 pts Continuity)
   Consider the function $f : \mathbb{R}^4 \to \mathbb{R}$ defined by $f((w,x,y,z)) = \langle (w,x),(y,z) \rangle$.
   For any $\epsilon > 0$ find $\delta > 0$ (which will depend on ϵ) so that if $|(w,x,y,z)| < \delta$ then $f((w,x,y,z)) < \epsilon$.
   This shows that the function f is continuous.

2. (20 pts Matrices) There is a sequence of elementary row operations which transforms the matrix

$$\begin{bmatrix} 2 & 0 & 2 & 0 \\ 1 & 2 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

to the reduced row eschelon form

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{bmatrix}.$$

Find (a, b, c).

3. (20 pts Topology) Prove either of the following two equivalent facts:

(a)
$$\left\{ \begin{bmatrix} a \\ b \end{bmatrix} | \langle (a,b),(2,3) \rangle \neq 0 \right\}$$
is open in $\mathbb{R}^2$.

(b)
$$\left\{ \begin{bmatrix} a \\ b \end{bmatrix} | \langle (a,b),(2,3) \rangle = 0 \right\}$$
is closed in $\mathbb{R}^2$.

4. (20 pts Compacta) Consider a continuous function $f : \mathbb{R}^2 \to \mathbb{R}$. Assume that if $|(x, y)| > 10$ then $f(x, y) = 0$.
Show that there is some point in $\mathbb{R}^2$ at which f takes its maximum value. Note that $\mathbb{R}^2$ is not compact.

5. (20 pts Derivatives)
Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ with $f((0,0)) = 0$ and otherwise

$$f((x,y)) = \frac{xy}{x^2 + y^2}.$$

This function is differentiable at every point except $(0,0)$ where it is not. Find the following or show they do not exist:

(a) the two by one total derivative matix $(Df)(1,2)$,

(b) $f'((0,0);(1,2))$,

(c) the two by one Jacobian matrix $\text{Jac}f((0,0))$.

6. (20 pts Class C^1) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$f((x,y)) = \begin{bmatrix} xy \\ x+y \end{bmatrix}.$$

(a) Compute all the directional derivatives of f.

(b) Use part (a) to show that f is continuously differentiable.
Recall the definition: A function is continuously differentiable if all parial derivatives $D_j f_i$ exist and are continuous.