Math 21A Fall 2014 Practice Final, December.

Please do not use any calculators, cell phones or books. You need not simplify your answers.

Basic derivatives:

1. \( \frac{d}{dx}(c) = 0 \)
2. \( \frac{d}{dx}(x^n) = nx^{n-1} \)
3. \( \frac{d}{dx}(e^x) = e^x \)
4. \( \frac{d}{dx}(a^x) = \ln(a) a^x \)
5. \( \frac{d}{dx}(\ln |x|) = \frac{1}{x} \)
6. \( \frac{d}{dx}(\log_a(x)) = \frac{1}{\ln(a)x} \)
7. \( \frac{d}{dx}(\sin(x)) = \cos(x) \)
8. \( \frac{d}{dx}(\cos(x)) = -\sin(x) \)
9. \( \frac{d}{dx}(\tan(x)) = \sec^2(x) \)
10. \( \frac{d}{dx}(\sec(x)) = \sec(x) \tan(x) \)
11. \( \frac{d}{dx}(\csc(x)) = -\csc(x) \cot(x) \)
12. \( \frac{d}{dx}(\cot(x)) = -\csc^2(x) \)
13. \( \frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}} \)
14. \( \frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}} \)
15. \( \frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2} \)
16. \( \frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{|x| \sqrt{x^2-1}} \)
17. \( \frac{d}{dx}(\csc^{-1}(x)) = -\frac{1}{|x| \sqrt{x^2-1}} \)
18. \( \frac{d}{dx}(\cot^{-1}(x)) = -\frac{1}{1+x^2} \)
19. \([f + g]' = f' + g' \)
20. \([fg]' = f'g + gf' \)
21. \( \frac{d}{dx} \left( \frac{g}{f} \right) = \frac{g'f - gf'}{f^2} \)
22. \( [f(g(x))]' = g'(x)f'(g(x)) \)
23. \( [f^{-1}](x) = \frac{1}{f'(f^{-1}(x))} \)
1. (8 points) Find $\frac{dy}{dx}$ for each of the following:
   
   (a) $y = (x^2 + 1)\sqrt{x}$
   
   (b) $y = \ln(\sec^2(x))$
   
   (c) $y = x^{(x^2)}$
   
   (d) $y = \tan^{-1}(\tan(x))$
   
   (e) $y = \frac{(x^3 - 1)^4}{\sqrt{x^2 - 1}}$
   
   (f) $\ln(y) + y = x$

   For this one the answer should be in terms of $y$.

2. (5 points) Find each of the following limits if it exists:
   
   (a) $\lim_{x \to 0^+} \left[ \tan^{-1}\left(\frac{\sin(x)}{x^2}\right) \right]$ 
   
   (b) $\lim_{x \to 0} \frac{\cos(x) - 1}{x \sin(x)}$
   
   (c) $\lim_{h \to 0} \frac{\ln(h + 1)}{h}$
   
   (d) $\lim_{x \to \infty} \frac{x^2 - 5x + \sin(x^2) - x^{-2}}{3x^2 + \cos(x^2 - 1)}$

3. (5 points) Assume that $f(2) = 5$, $f'(2) = 3$, $\lim_{x \to \infty}(f(x)) = 7$
   
   (a) Find $h'(2)$ if $h(x) = f^2(x)$ is the square of $f(x)$.
   
   (b) Find $g'(5)$ if $g(x) = f^{-1}(x)$ is the inverse function to $f(x)$.
   
   (c) Find $\lim_{x \to 2} [f((f(x) - 5)^{-2}) - f(x)]$. 


4. (5 points) Find the area of the largest rectangle with one edge along the $x$-axis which fits in the region above the $x$ axis and below the curve $y = 3 - x^2$.

5. (5 points) A conical water tank is 20 feet tall, 5 feet across at the top and leaking at a rate of 3 cubic feet per minute. Find the speed at which the surface level of the water is dropping when it is 5 feet deep.

6. (7 points) Consider the function $f(x) = 2x^3 - 3x^2 + 2$

   (a) Use one step of Newton’s method to get an estimate $x_1$ for a value at which the function is zero starting with $x_0 = -1$.

   (b) Find the linearization $L(x)$ of $f(x)$ at $a = -1$ and the point at which $L(x) = 0$.

   (c) Find the minimum and maximum values of $f(x)$ in the interval $[-1, 1]$.

   (d) Find the interval(s) in $[-1, 1]$ in which $f(x)$ is concave up.