Math 21C
Kouba
Problems Using (*) and (**)(*) from the Integral Test Handout

1.) The series \( \sum_{n=1}^{\infty} \frac{1}{n} \) diverges. Use (*) to put a lower and an upper bound on the partial sum
   a.) \( s_{10} \), the sum of the first 10 terms of this series.
   b.) \( s_{10000} \), the sum of the first 1000 terms of this series.
   c.) \( s_{1,000,000} \), the sum of the first 1,000,000 terms of this series.

2.) The series \( \sum_{n=1}^{\infty} \frac{1}{n^2} \) converges.
   a.) Compute the partial sum \( s_{10} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{10^2} = \sum_{i=1}^{10} \frac{1}{i^2} \).
   b.) Use (*)(**) to put a lower and an upper bound on the error (remainder) \( R_{10} = \frac{1}{11^2} + \frac{1}{12^2} + \frac{1}{13^2} + \cdots \) for the partial sum \( s_{10} \).
   c.) Use (*)(**) to put a lower and an upper bound on the error (remainder) \( R_{100} = \frac{1}{101^2} + \frac{1}{102^2} + \frac{1}{103^2} + \cdots \) for the partial sum \( s_{100} \).
   d.) What should \( n \) be so that the partial sum \( s_n = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} = \sum_{i=1}^{n} \frac{1}{i^2} \)
estimates the exact value of \( \sum_{n=1}^{\infty} \frac{1}{n^2} \) with an error \( R_n \) of at most 0.0001?

3.) The series \( \sum_{n=1}^{\infty} (2/3)^{n-1} \) converges.
   a.) Compute the partial sum \( s_{10} = 1 + (2/3) + (2/3)^2 + \cdots + (2/3)^9 = \sum_{i=1}^{10} (2/3)^{i-1} \).
   b.) Use (*)(**) to put a lower and an upper bound on the error (remainder) \( R_{10} = (2/3)^{10} + (2/3)^{11} + (2/3)^{12} + (2/3)^{13} + \cdots \).
   c.) Compute the exact value of \( R_{10} = (2/3)^{10} + (2/3)^{11} + (2/3)^{12} + (2/3)^{13} + \cdots \).
   d.) What should \( n \) be so that the partial sum \( s_n = 1 + (2/3) + (2/3)^2 + \cdots + (2/3)^{n-1} = \sum_{i=1}^{n} (2/3)^{i-1} \) estimates the exact value of \( \sum_{n=1}^{\infty} (2/3)^{n-1} \) with an error \( R_n \) of at most 0.0001?
   e.) What is the exact value of the series \( \sum_{n=1}^{\infty} (2/3)^{n-1} \)?