Math 21C
Kouba
The Comparison Tests and Limit Comparison Tests

COMPARISON TESTS: Assume that sequences a_n and b_n satisfy $0 \le a_n \le b_n$. The following statements can now be made.

I.) If the series $\sum_{n=1}^{\infty} b_n$ converges, then the series $\sum_{n=1}^{\infty} a_n$ converges.

II.) If the series $\sum_{n=1}^{\infty} a_n$ diverges, then the series $\sum_{n=1}^{\infty} b_n$ diverges.

LIMIT COMPARISON TESTS: Assume that we know what the series $\sum_{n=1}^{\infty} b_n$ does (converge or diverge) and we are trying to determine what the series $\sum_{n=1}^{\infty} a_n$ does. Assume also that $a_n > 0$, $b_n > 0$, and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$, where L is a *positive, finite* number.

I.) If the series $\sum_{n=1}^{\infty} b_n$ converges, then the series $\sum_{n=1}^{\infty} a_n$ converges.

II.) If the series $\sum_{n=1}^{\infty} b_n$ diverges, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

THE FOLLOWING ARE SPECIAL CASES FOR THE LIMIT COMPARISON TEST.

III.) a.) If $L = 0$ and the series $\sum_{n=1}^{\infty} b_n$ converges, then the series $\sum_{n=1}^{\infty} a_n$ converges.

b.) If $L = 0$ and the series $\sum_{n=1}^{\infty} b_n$ diverges, then NO CONCLUSION can be made about the series $\sum_{n=1}^{\infty} a_n$ using this test.

IV.) a.) If $L = \infty$ and the series $\sum_{n=1}^{\infty} b_n$ diverges, then the series $\sum_{n=1}^{\infty} a_n$ cdiverges.

b.) If $L = \infty$ and the series $\sum_{n=1}^{\infty} b_n$ converges, then NO CONCLUSION can be made about the series $\sum_{n=1}^{\infty} a_n$ using this test.