

Math 21B Kouba
Differentials

RECALL: (from Math 21A)

If $Y = f(X)$ and x -values: $a \rightarrow a+h$, then exact change of f is

$$\Delta f = f(a+\Delta x) - f(a);$$

the differential of f is

$$\boxed{df = f'(a) \cdot \Delta x};$$

if Δx is small, then $df \approx \Delta f$.

Definition: If $Z = f(X, Y)$ and

x -values: $a \rightarrow a + \Delta x$

y -values: $b \rightarrow b + \Delta y$; then

exact change of f is

$$\Delta f = f(a+\Delta x, b+\Delta y) - f(a, b);$$

the differential of f is

$$\boxed{df = f_x(a, b) \cdot \Delta x + f_y(a, b) \cdot \Delta y}$$

$$= (f_x(a, b), f_y(a, b)) \cdot (\Delta x, \Delta y)$$

$$= \vec{\nabla} f(a, b) \cdot \frac{(\Delta x, \Delta y)}{ds} \cdot ds$$

$$= \vec{\nabla} f(a, b) \cdot \vec{u} \cdot ds, \text{ i.e.}$$

$$\boxed{df = D_{\vec{u}} f(a, b) \cdot ds};$$

if ds is small, then $df \approx \Delta f$.

