

Math 21C
 Kouba
 Discussion Sheet 1

1.) Compute the first five terms (starting with $n=1$) of each sequence. Determine whether each sequence converges or diverges.

- a.) $\{3\}$ b.) $\{3^n\}$ c.) $\left\{\frac{3}{n}\right\}$ d.) $\left\{\left(\frac{1}{3}\right)^n\right\}$
 e.) $\{3^{1/n}\}$ f.) $\left\{\frac{n+5}{n+2}\right\}$ g.) $\{n(3-n)\}$ h.) $\left\{\frac{n^3+n^2-n+7}{4n^3+5n^2-2}\right\}$
 i.) $\{(0.9999)^n\}$ j.) $\{(1.00001)^n\}$ k.) $\left\{\left(\frac{-2}{3}\right)^n\right\}$ l.) $14/3, 15/5, 16/7, 17/9, \dots$
 m.) $\left\{\frac{\sin 3n}{n}\right\}$ n.) $\left\{\left(\frac{\sqrt{7}}{\ln 14}\right)^n\right\}$ o.) $\{\cos(2n\pi)\}$ p.) $\{(1+1/n)^n\}$
 q.) $\left\{\frac{3^n}{n!}\right\}$ r.) $\{\sin(\pi/2 + n\pi)\}$ s.) $\left\{\frac{1000^n}{n!}\right\}$ t.) $\left\{\frac{n^2}{e^n}\right\}$
 u.) $\{(n-1)(n-2)(n-3)(n-4)(n-5)\}$ v.) $\{3 + (-1)^n\}$ w.) $\left\{\sum_{i=1}^n i^2\right\}$
 x.) $\left\{\sum_{i=1}^n (2/3)^{i-1}\right\}$ y.) $\left\{\sum_{i=1}^n (1+i/n)^3(1/n)\right\}$

2.) Find a formula (starting with $n = 1$) for each of the following sequences.

- a.) 4, 8, 12, 16, 20, 24, ... b.) 4, 6, 8, 10, 12, 14, ...
 c.) 4, -9, 16, -25, 36, -49, 64, ... d.) 2, 12, 30, 56, 90, 132, ...
 e.) 2, 12, 2, 12, 2, 12, ... f.) $\frac{1}{2}, \frac{0}{6}, \frac{1}{12}, \frac{4}{20}, \frac{9}{30}, \frac{16}{42}, \dots$

3.) Determine whether the following series converge or diverge.

- a.) $\sum_{n=0}^{\infty} 2^n$ b.) $\sum_{n=1}^{\infty} 2$ c.) $\sum_{n=2}^{\infty} \frac{1}{2^n}$ d.) $\sum_{n=1}^{\infty} 0.000001$
 e.) $\sum_{n=1}^{\infty} (0.98)^{n+3}$ f.) $\sum_{n=1}^{\infty} 5(-2/3)^{n-1}$ g.) $\sum_{n=0}^{\infty} (-1)^n$ h.) $\sum_{n=0}^{\infty} (1/4)(3/2)^{n+3}$
 i.) $\sum_{n=1}^{\infty} \frac{n+2}{n+1000}$ j.) $\sum_{n=2}^{\infty} (1-1/n)^n$ k.) $\sum_{n=1}^{\infty} \cos n\pi$
 l.) $\sum_{n=7}^{\infty} \left(\frac{1}{\ln n} - \frac{1}{\ln(n+1)}\right)$ m.) $\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6}$ HINT: Use partial fractions first.

4.) Compute the exact value of the following convergent series :

$$6 - 2 + \frac{2}{3} - \frac{2}{9} + \frac{2}{27} - \frac{2}{81} + \dots$$

5.) Find the 25th term in the following sequence : 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

6.) Find the sum of the first 200 terms for each of the following sequences.

a.) 3, 7, 11, 15, 19, 23, ... a.) 9, 25, 49, 81, 121, 169, ...

7.) A ball bearing is dropped from a building 200 feet high. Each time the ball bearing rebounds to 40% of its falling distance. What is the total distance the ball bearing will travel ?

8.) Determine the limit of the following sequence :

$$2, \quad 2 - \frac{1}{2}, \quad 2 - \frac{1}{2 - \frac{1}{2}}, \quad 2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2}}}, \quad 2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2}}}} \quad \dots$$

9.) Start at the origin and move 10 units along the positive y-axis. Turn 90 degrees to the right and move 70% of that distance. Turn 90 degrees to the right and move 70% of that distance. Turn 90 degrees to the right and move 70% of that distance. Turn 90 degrees to the right and move 70% of that distance. Continue this process. At what point (x,y) will you "end" ?

"The mind is not a vessel to be filled, but a fire to be ignited." – Plutarch