

Math 21C  
 Kouba  
 Discussion Sheet 5

1.) Write each power series as an ordinary function.

$$\begin{array}{lll} \text{a.) } \sum_{n=5}^{\infty} x^n & \text{b.) } \sum_{n=0}^{\infty} 2^n x^n & \text{c.) } \sum_{n=0}^{\infty} \frac{(-3)^{n+1} x^n}{5^{n-1}} \\ \text{d.) } \sum_{n=4}^{\infty} n x^{n-1} & \text{e.) } \sum_{n=0}^{\infty} n^2 x^{n-1} & \text{f.) } \sum_{n=1}^{\infty} \frac{x^{n+3}}{n} \\ \text{g.) } \sum_{n=1}^{\infty} (-1)^n \frac{x^n}{2^n n!} & \text{h.) } \sum_{n=2}^{\infty} (-1)^n \left(\frac{2}{5}\right)^{2n} \frac{x^{2n+1}}{2n+1} & \end{array}$$

2.) Use any method to find the exact value of each of the following convergent series.

$$\begin{array}{lll} \text{a.) } \sum_{n=0}^{\infty} 3 \left(\frac{-2}{3}\right)^n & \text{b.) } \sum_{n=4}^{\infty} \frac{(-1)^{n+2}}{2^{n-3}} & \text{c.) } \sum_{n=1}^{\infty} n^2 \left(\frac{1}{2}\right)^n \\ \text{d.) } \sum_{n=0}^{\infty} n(n-1) \left(\frac{3}{4}\right)^{n+1} & \text{e.) } \sum_{n=0}^{\infty} \frac{(\ln 2)^n}{n!} & \text{f.) } \sum_{n=2}^{\infty} (-1)^n \frac{9^n}{(2n)!} \end{array}$$

3.) Find the distance between the points  $(3, -2, 4)$  and  $(2, -6, -4)$ .

4.) Find an equation of the sphere whose diameter has endpoints  $(2, 4, -5)$  and  $(0, -2, 4)$ .

5.) Find the center and radius of the following sphere:  $x^2 + y^2 + z^2 = 2x - 4y + 6z - 5$

6.) Determine a formula (and sketch the surface) for the set of all points  $(x, y, z)$  in three-dimensional space which are

- 4 units from the point  $(2, -3, 0)$ .
- 3 units from the  $z$ -axis.
- $1/2$  unit from the  $x$ -axis.
- 2 units from the plane  $y = 3$ .
- equidistant from the points  $(3, 0, 0)$  and  $(0, 0, 3)$ .
- equidistant from the planes  $z = 2$  and  $z = 6$ .
- equidistant from the planes  $x = 3$  and  $y = 2$ .
- equidistant from the point  $(0, 0, 2)$  and the  $xy$ -plane.

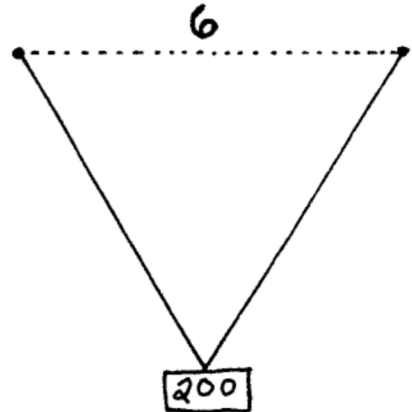
7.) a.) If vector  $\vec{A} = \overrightarrow{(1, 0, -2)}$ , then what is the unit vector in the same direction as  $\vec{A}$  ?  
 b.) If vector  $\vec{A} = \overrightarrow{(a, b, c)}$ , and  $a, b,$  and  $c$  are not all zero, then what is the unit vector in the same direction as  $\vec{A}$  ?

8.) Determine the vector  $\vec{B}$ , which starts at point  $(1, -1, 0)$  and ends at point  $(-1, 2, 6)$ . Find a vector of length 2 pointing in the opposite direction of  $\vec{B}$ .

9.) Find two vectors of length 3 which are both perpendicular to

- a.) vector  $\vec{W} = 3\vec{i} + 4\vec{j}$  .  
 b.) vector  $\vec{W} = \vec{i} - 2\vec{j} + 2\vec{k}$  .
- 10.) A sailboat starts at the origin (0,0), then sails  
 a.) 3 km NW, then turns and sails  
 b.) 2 km  $60^\circ$  North of East, then turns and sails  
 c.) 4 km SE, then turns and sails  
 d.) 10 km  $30^\circ$  South of West, and stops. What are the sailboat's coordinates now ?

11.) Two strong wires of equal length are hung from two supports which are at the same height and 6 feet apart. Each wire is attached to the same point of a 200 pound weight. What is the force of tension (in pounds) on each wire if the wires are each



- a.) 5 feet long ?  
 b.) 20 feet long ?  
 c.) 3.1 feet long ?  
 d.) 3.01 feet long ?

12.) You can swim at a constant speed of 5 mph. You wish to swim across a river 1 mile wide and land at a point directly across the river from where you start swimming. If the river flows at the constant rate of 3 mph, in what direction should you swim to accomplish this ? How long will it take you to swim across the river ?

\*\*\*\*\* The following problem is for recreational purposes only. \*\*\*\*\*

13.) A circus is witnessed by 120 people who have paid a total of \$120. The women paid \$5 each, the men paid \$2 each, and the children paid 10 cents each. How many women and children went to the circus ?