

Math 21C  
Kouba  
Discussion Sheet 7

1.) Graph each of the following equations in two-dimensional space.

a.)  $y = 3$       b.)  $x = -2$       c.)  $y = x$       d.)  $y = 3 - x$       e.)  $y = x^3$   
f.)  $y = e^x$       g.)  $y = \ln x$       h.)  $y = \sqrt{x}$       i.)  $x = y^2$       j.)  $y = \frac{1}{x}$

2.) Sketch the level curves for each of the following equations (surfaces) using the following values of  $z$  : -3, -2, -1, 0, 1, 2, 3

a.)  $z = y$       b.)  $z = 1 - x - y$       c.)  $z^2 = x^2 + y^2$       d.)  $x^2 + y^2 + z^2 = 9$

3.) Sketch all three coordinate plane traces (i.e.,  $x = 0$ ,  $y = 0$ , and  $z = 0$ ) for each of the following equations (surfaces).

a.)  $x + 2y + 3z = 6$       b.)  $z = x^2 + y^2$       c.)  $z = y^2 - x^2$       d.)  $z^2 = x^2 + y^2$

4.) Sketch in three-dimensional space each of the following equations (surfaces). Use intercepts, traces, and/or level curves, if necessary.

a.)  $y = 3$       b.)  $x = -2$       c.)  $y = x$       d.)  $y = 3 - x$       e.)  $y = x^3$   
f.)  $y = e^x$       g.)  $y = \ln x$       h.)  $y = \sqrt{x}$       i.)  $x = y^2$       j.)  $y = \frac{1}{x}$   
k.)  $x^2 + y^2 + z^2 = 4$       l.)  $x + 2y + 3z = 6$       m.)  $z = x^2 + y^2$       n.)  $z^2 = x^2 + y^2$   
o.)  $z^2 = x^2 + y^2 - 1$       p.)  $z^2 = x^2 + y^2 + 1$       q.)  $z = y^2 - x^2$

5.) a.) Consider the graph of  $y = \ln(x - 1)$  in the  $xy$ -plane. Find an equation for the surface created by revolving this graph about the

i.)  $x$ -axis .      ii.)  $y$ -axis .

b.) Consider the graph of  $z = \sin x$  in the  $xz$ -plane. Find an equation for the surface created by revolving this graph about the

i.)  $x$ -axis .      ii.)  $z$ -axis .

6.) Determine and sketch the domain of each function in 2D-Space and find the range of each function.

a.)  $z = 1 + x^2 + y^2$       b.)  $z = 1 - x^2 - y^2$   
c.)  $z = 1 - x^2 + y^2$       d.)  $z = 1 - x - y$   
e.)  $f(x, y) = \sqrt{1 - x - y}$       f.)  $f(x, y) = \sqrt{1 - x^2 - y^2}$   
g.)  $f(x, y) = 5 + e^{-x^2 - y^2}$       h.)  $f(x, y) = 3 - \sqrt{y - \ln x}$   
i.)  $z = 3 \cos x + 4 \sin y$       j.)  $z = 2 - 5 \sin(\ln y)$

$$\begin{array}{ll}
\text{k.) } z = \ln(25 - x^2 - y^2) & \text{l.) } f(x, y) = \ln(x^2 + y^2 - 25) \\
\text{m.) } f(x, y) = \frac{7}{x^2 - y} & \text{n.) } f(x, y) = \frac{7}{x^2 + y^2} \\
\text{o.) } f(x, y) = \frac{8}{2 + \sqrt{x - 2y}} & \text{p.) } f(x, y) = \frac{8}{2 - \ln(x + y)}
\end{array}$$

7.) Determine and sketch the domain of each function in 2D-Space.

$$\begin{array}{ll}
\text{a.) } f(x, y) = \ln(x^2 + y^2 - 4) & \text{b.) } f(x, y) = \ln(1 + x + y) \\
\text{c.) } f(x, y) = \frac{1}{4 - \sqrt{25 - x^2 - y^2}} & \text{d.) } f(x, y) = \sqrt{(x^2 - 4)(y^2 - 1)}
\end{array}$$

\*\*\*\*\* The following problem is for recreational purposes only. \*\*\*\*\*

8.) A snail is at the bottom of a well which is 100 feet deep. Each day the snail climbs up 7 feet and down 5 feet. In how many days will the snail reach the top of the well ?