1. Determine whether the following limit exists.

$$\lim_{n \to \infty} \frac{10^6n}{\sqrt{n}^n}$$

2. How many terms of the following convergent series should be used to estimate its value with an error of at most 0.1?

$$\sum_{n=1}^{\infty} 2ne^{-n^2}$$

3. Use the alternating series error bound to estimate the error from using the first two nonzero terms in the Maclaurin series for $e^{-x^2}$ to estimate the definite integral

$$\int_0^{\frac{\pi}{2}} e^{-x^2} \, dx.$$ 

4. Determine whether the following series converges.

$$\sum_{n=1}^{\infty} \frac{1}{\ln(n)\sqrt{n}}$$

5. Determine the values of $x$ for which the following series converges. Be sure to check the end points of the interval.

$$\sum_{n=0}^{\infty} \frac{(2-x)^n}{\sqrt{n}}$$

6. Find the first three nonzero terms of the Taylor series about $x = 0$ for the following function.

$$\frac{1 + x^2}{1 - 2x}$$

7. Find an equation for the set of points in space which are the same distance from the origin as they are from the point $(1, 1, 1)$.

8. Find the cosine of any one of the angles in the triangle with corners at $(1, 1, 1)$, $(1, 2, 3)$ and $(5, 4, 3)$. 
9. Find an equation for the plane containing the points (1, 1, 1), (1, 2, 3) and (5, 4, 3).

10. If the distance between the points \( P \) and \( Q \) is 5 and the distance between the point \( P \) and the line \( Q + t \mathbf{u} \) is 4 what is the distance between the point \( P \) and the plane through \( Q \) and perpendicular to \( \mathbf{u} \)?

11. Show that the following limit does not exist.

\[
\lim_{(x, y) \to (1, 0)} \frac{xy - y}{(x - 1)^2 + y^2}
\]

12. Find \( \frac{dw}{ds} \) at \( s = \frac{\pi}{2} \) if \( w = x^2 y^2, y(s) = \sin(s), x(s) \) is unknown but \( x(1) = 3 \) and \( \frac{dx}{ds} = 2 \) at \( s = 1 \).

13. Find the two directions in which the directional derivative of \( f(x, y) \) at the point \( (1, 1) \) is zero.

\[
f(x, y) = x^2 y - y^2 x
\]

14. Find the point on the elliptic paraboloid given by \( z = 1 - x^2 - y^2 \) which is closest to the point \( (0, 0, 2) \).

15. Find and classify the critical point(s) and find the maximum value of the function \( y^2 - x^2 \) on the circular disk given by \( 4(x - 1)^2 + 4y^2 \leq 9 \).

16. Find the largest area of a rectangle with corners on the ellipse \( \frac{x^2}{10} + y^2 = 1 \) and edges parallel to the \( x \) and \( y \) axes.

17. (Optional extra credit problem.) Find the value of \( a \) for which the following limit exists and find the limit.

\[
\lim_{x \to 0} \frac{\sin(ax) - \sin(x) - x}{x^3}
\]