To receive full credit you must show all of your work. Please do not use any cell phones, notes or books. A calculator is allowed. Do not simplify your solutions.
1. Determine whether the following series converge. Specify the convergence tests you use.

(a) \[ \sum_{n=1}^{\infty} \frac{\cos^2(n)}{n^2 + \sqrt{n}} \]

(b) \[ \sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{n} \]
2. For each of the following find an upper bound for the error resulting from estimating the infinite sum with just the first 5 terms.

(a) \[
\sum_{n=1}^{\infty} \frac{2n}{(1 + n^2)^2}
\]

(b) \[
\sum_{n=1}^{\infty} (-1)^n \frac{3}{2^{2n}}
\]
3. Determine the values of $x$ for which the following series converges. Be sure to check the end points of the interval.

$$\sum_{n=1}^{\infty} \sqrt{n} \frac{x^n}{5^n}$$

4. Find the first three nonzero terms of the Taylor series about $x = 0$ for the following function.

$$f(x) = \cos(2x) - x \sin(x)$$
5. Find the ground speed (magnitude of the velocity vector) of a fly if the wind is blowing the fly 4 \text{ mi/hr} northeast while the fly is flying 3 \text{ mi/hr} west.

6. Find an equation for the plane through the points (1, 1, 1), (1, 2, 3) and (−1, 0, 3).
7. Consider the function \( f(x, y) = 3\sqrt{4 - x^2 - y^2} \).

(a) Find and sketch the domain of \( f \).

(b) Find and sketch the range of \( f \).

(c) Describe the surface \( z = f(x, y) \).
8. Consider again the surface $z = 3\sqrt{4 - x^2 - y^2}$. Find a parametric equation for the line normal to the surface at the point with $x = y = 1$. 

9. Laplace’s equation for heat in a plate is satisfied by $f(x, y)$ if $f_{xx} + f_{yy} = 0$. Determine whether each of the following satisfy Laplace’s equation.

(a) $f(x, y) = e^{-2y} \cos(3x)$

(b) $f(x, y) = \ln(x^2 + y^2)$
10. Find all the local maxima, local minima and saddle points of the function

\[ f(x, y) = x^3 - y^3 - 2xy + 6. \]
11. Find the maximum value of the function $f(x, y, z) = x - 2y + 3z$ on the sphere $x^2 + y^2 + z^2 = 14$. 
12. (Optional extra credit problem.) Evaluate the sum

\[ \sum_{n=0}^{\infty} \frac{1}{(4n)!} \]