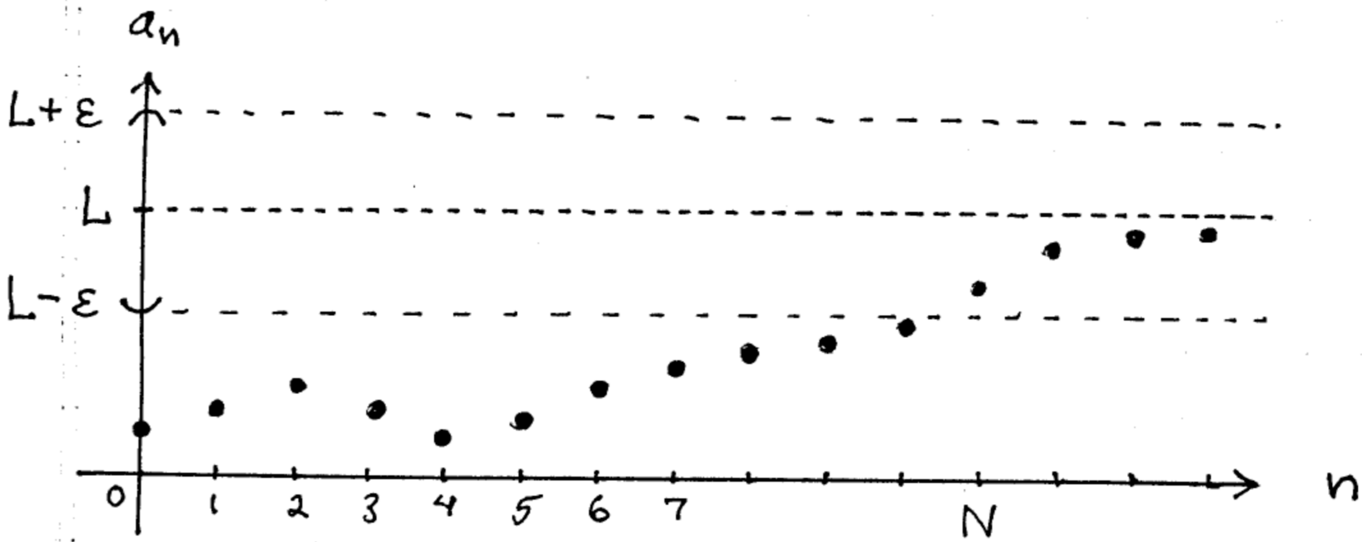
Math 21C
Koubа
Formal Definition of Limit
of a Sequence

<u>Definition</u>: $\lim_{n \to \infty} a_n = L$ (a finite #)

means: For each number $\varepsilon > 0$ there exists an integer N so that if $n > N$, then $|a_n - L| < \varepsilon$.



<u>Note</u>: $|a_n - L| < \varepsilon \longrightarrow$

$-\varepsilon < a_n - L < \varepsilon \longrightarrow$

$L - \varepsilon < a_n < L + \varepsilon$

Example: Prove that $\lim_{n\to\infty} \frac{n}{n+2} = 1$.

Proof: Let $\varepsilon > 0$ be given. Find an integer N so that

if $n > N$, then $\left|\frac{n}{n+2} - 1\right| < \varepsilon$.

Begin with $\left|\frac{n}{n+2} - 1\right| < \varepsilon$ and solve for n. Then

$$\left|\frac{n}{n+2} - 1\right| < \varepsilon \text{ iff } \left|\frac{n}{n+2} - \frac{n+2}{n+2}\right| < \varepsilon$$

$$\text{iff } \left|\frac{n-n-2}{n+2}\right| < \varepsilon$$

$$\text{iff } \left|\frac{-2}{n+2}\right| < \varepsilon$$

$$\text{iff } \frac{|-2|}{|n+2|} < \varepsilon$$

$$\text{iff } \frac{2}{|n+2|} < \varepsilon$$

$$\text{iff } \frac{2}{n+2} < \varepsilon \quad (\text{Since } n \to \infty,\ n \text{ is} +)$$

$$\text{iff } \frac{2}{\varepsilon} < n+2 \text{ iff } n > \frac{2}{\varepsilon} - 2.$$

Now CHOOSE N to be any integer $N \geq \frac{2}{\varepsilon} - 2$. Thus, if $n > N$, it follows that $\left|\frac{n}{n+1} - 1\right| < \varepsilon$. QED