

Math 21C Midterm I Friday, April 19 Spring 2024

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Student ID:

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You may not use a calculator.  
You may use one page of notes.  
You may not use the textbook.  
Please do not simplify answers.

1. (9 pts each: Series)

Determine for each part whether the series converges or diverges. Write clear and complete solutions including the name of the series test you use and what your answer is.

(a)

$$\sum_{n=1}^{\infty} \frac{3n^2 + 1}{n^2 + n}$$

(b)

$$\sum_{n=1}^{\infty} \left(\frac{-3}{2}\right)^n$$

(c)

$$\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^3+5}}$$

(d)

$$\sum_{n=1}^{\infty} (-1)^n \sqrt{\frac{n+1}{n^3+5}}$$

(e)

$$\sum_{n=1}^{\infty} n e^{-n^2}$$

(f)

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$$

(g)

$$\sum_{n=1}^{\infty} \left( e^{\frac{1}{n}} - e^{\frac{1}{n+1}} \right)$$

(h)

$$\sum_{n=1}^{\infty} \left( 1 - \frac{1}{n} \right)^n$$

2. (10 pts: Story)

A redwood tree increases in diameter each spring. Each spring its diameter grows 99 percent as much as it did the previous spring. During its first spring its diameter grows to one foot.

What will be the eventual diameter of the tree if it lives forever?

3. (9 pts: Integral Errors)

The series

$$T = \sum_{n=1}^{\infty} ne^{-n^2}$$

converges rapidly.

(a) Find any upper and lower bounds for  $T$ .

(b) Find upper and lower bounds for  $T$  which differ by at most  $\frac{1}{2}$ .

4. (9 pts: Alternating Errors)

The alternating series

$$S = \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$$

converges slowly.

(a) Find any upper and lower bounds for  $S$ .

(b) Find upper and lower bounds for  $S$  which differ by at most  $\frac{1}{2}$ .



5. (10 pts: Extra Credit... you may skip this problem)  
You know that  $\sum_{m=0}^{\infty} 7r^m = 8$ . Find the exact value of

$$\sum_{m=0}^{\infty} 7\sqrt[3]{r^m}.$$